

# Algebra pellach a calcalws

FP2



1. Defnyddiwch yr amnewid  $x = y^2$  i enrhifo'r integrynn

$$\int_1^4 \frac{dx}{\sqrt{x(9-x)}},$$

gan roi eich ateb yn gywir i ddau ffigur ystyrlon.

[6]



2007

1.  $x = y^2 \Rightarrow dx = 2y dy$  and  $[1, 4] \rightarrow [1, 2]$

B1B1

$$I = \int_1^2 \frac{2y dy}{\sqrt{y^2(9-y^2)}}$$

M1

$$= 2 \int_1^2 \frac{dy}{\sqrt{9-y^2}}$$

A1

$$= 2 \left[ \sin^{-1} \left( \frac{y}{3} \right) \right]_1^2$$

A1

$$= 0.78$$

A1



2007

3. (a) Using the substitution  $u = x^2$ , evaluate the integral

$$\int_0^{\sqrt{3}} \frac{x dx}{(9 + x^4)},$$

giving your answer in the form  $\frac{\pi}{k}$ , where  $k$  is an integer.

[5]

- (b) Evaluate the integral

$$\int_0^1 \frac{dx}{\sqrt{25 - 9x^2}}.$$

[4]



2008

3 (a)  $u = x^2 \Rightarrow du = 2x dx$  B1  
and  $[0, \sqrt{3}] \rightarrow [0, 3]$  B1

$$I = \frac{1}{2} \int_0^3 \frac{du}{9+u^2}$$
 M1

$$= \frac{1}{6} \left[ \arctan\left(\frac{u}{3}\right) \right]_0^3$$
 A1

$$= \frac{\pi}{24}$$
 A1

(b)  $\int_0^4 \frac{dx}{\sqrt{25 - 9x^2}} = \frac{1}{3} \int_0^4 \frac{dx}{\sqrt{25/9 - x^2}}$  M1

$$= \frac{1}{3} \left[ \sin^{-1}\left(\frac{3x}{5}\right) \right]_0^1$$
 A1

$$= \frac{1}{3} \sin^{-1} 0.6$$
 A1

$$= 0.2145$$
 A1

[FT on minor arithmetic error, do not FT the omission of the factor 1/3]



2008

2. Using the substitution  $u = \tan x$ , evaluate the integral

$$\int_0^{\frac{\pi}{6}} \frac{\sec^2 x}{\sqrt{3 - \sec^2 x}} dx .$$

Explain briefly why the integral could not be evaluated if the upper limit were changed to  $\frac{\pi}{3}$ . [7]



2009

2.

$$u = \tan x \Rightarrow du = \sec^2 x dx$$

and  $[0, \pi/6] \rightarrow [0, 1/\sqrt{3}]$

B1

$$I = \int_0^{1/\sqrt{3}} \frac{du}{\sqrt{3 - (1 + u^2)}}$$

B1

M1

[Must be correct here]

$$= \int_0^{1/\sqrt{3}} \frac{du}{\sqrt{2 - u^2}}$$

A1

$$= \left[ \sin^{-1} \left( \frac{u}{\sqrt{2}} \right) \right]_0^{1/\sqrt{3}}$$

A1

$$= 0.421$$

A1

Any valid reason, eg the denominator would be the square root of a negative number towards the upper limit.

B1



# 2009

5. The function  $f$  is defined by

$$f(x) = \frac{1}{(x+1)(x+2)(x+3)} .$$

(a) Express  $f(x)$  in partial fractions. [4]

(b) Evaluate the integral

$$\int_0^5 f(x) dx ,$$

giving your answer in the form  $\ln\left(\frac{m}{n}\right)$  where  $m, n$  are integers. [5]



2009

5. (a) Let  $\frac{1}{(x+1)(x+2)(x+3)} \equiv \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{x+3}$   
 $= \frac{A(x+2)(x+3) + B(x+1)(x+3) + C(x+1)(x+2)}{(x+1)(x+2)(x+3)}$  M1

$x = -1$  gives  $A = 1/2$ ;  $x = -2$  gives  $B = -1$ ;  $x = -3$  gives  $C = 1/2$  A1A1A1

(b)  $I = \left[ \frac{1}{2} \ln(x+1) - \ln(x+2) + \frac{1}{2} \ln(x+3) \right]_0^5$  B1B1  
 [B1 for 1 error]

$$= \frac{1}{2} (\ln 6 - \ln 49 + \ln 8 + \ln 4 - \ln 3) \quad \text{M1}$$

$$= \frac{1}{2} \ln \left( \frac{6 \times 8 \times 4}{49 \times 3} \right) \quad \text{A1}$$

$$= \ln \left( \frac{8}{7} \right) \quad \text{A1}$$



2009

1. Gan ddefnyddio'r amnewid  $u = x \sqrt{x}$ , enrhifwch yr integrynn

$$\int_0^2 \frac{\sqrt{x}}{\sqrt{9-x^3}} dx.$$

Rhowch eich ateb yn gywir i dri lle degol.

[5]



2010

1.  $u = x\sqrt{x} \Rightarrow du = \frac{3}{2}\sqrt{x}dx$  B1

and  $[0,2] \rightarrow [0, 2\sqrt{2}]$  B1

$$I = \frac{2}{3} \int_0^{2\sqrt{2}} \frac{du}{\sqrt{9-u^2}}$$
 M1

$$= \frac{2}{3} \left[ \sin^{-1}\left(\frac{u}{3}\right) \right]_0^{2\sqrt{2}}$$
 A1

$$= 0.821$$
 A1



2010

4. Mae'r ffwythiant  $f$  wedi'i ddiffinio gan

$$f(x) = \frac{3x^2}{(x+2)(x^2+2)}.$$

(a) Mynegwch  $f(x)$  yn nhermau ffracsynau rhannol. [4]

(b) Enrhifwch yr integrynn

$$\int_1^2 f(x) \, dx.$$

[6]



2010

4. (a) Let  $\frac{3x^2}{(x+2)(x^2+2)} \equiv \frac{A}{x+2} + \frac{Bx+C}{x^2+2}$

$$= \frac{A(x^2+2)+(Bx+C)(x+2)}{(x+2)(x^2+2)}$$

$$A = 2, B = 1, C = -2 \quad \text{M1}$$

A1A1A1

(b)  $I = 2 \int \frac{dx}{x+2} + \int \frac{x dx}{x^2+2} - 2 \int \frac{dx}{x^2+2}$  M1

$$= 2[\ln(x+2)] + \frac{1}{2}[\ln(x^2+2)] - \frac{2}{\sqrt{2}} \left[ \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) \right]^2 \quad \text{A1A1A1}$$

$$= 2 \ln 4 - 2 \ln 3 + \frac{1}{2}(\ln 6 - \ln 3) - \frac{2}{\sqrt{2}} \left( \tan^{-1}\left(\frac{2}{\sqrt{2}}\right) - \tan^{-1}\left(\frac{1}{\sqrt{2}}\right) \right) \quad \text{A1}$$

$$= 0.441 \quad \text{cao} \quad \text{A1}$$



2010

1. Gan ddefnyddio'r amnewid  $u = \sqrt{x}$ , enrhifwch yr integrynn

$$\int_1^4 \frac{1}{(9+x)\sqrt{x}} \, dx.$$

Rhowch eich ateb yn gywir i bedwar lle degol.

[5]



2011

1.  $u = \sqrt{x} \Rightarrow du = \frac{1}{2} \frac{1}{\sqrt{x}} dx$  B1

and  $[1,4] \rightarrow [1, 2]$  B1

$$I = 2 \int_1^2 \frac{du}{9 + u^2}$$
 M1

$$= \frac{2}{3} \left[ \tan^{-1} \left( \frac{u}{3} \right) \right]_1^2$$
 A1

$$= 0.1775$$
 A1



2011

8. Mae'r ffwythiant  $f$  wedi'i ddiffinio gan

$$f(x) = \frac{(x+1)^2}{(x-1)(x-2)} .$$

(a) Profwch ei bod yn bosibl ysgrifennu  $f(x)$  yn y ffurf

$$1 - \frac{4}{x-1} + \frac{9}{x-2} .$$

Trwy hyn, darganfyddwch fynegiadau ar gyfer  $f'(x)$  a  $f''(x)$ .

[7]



2011

8. (a) EITHER

$$f(x) = \frac{x^2 + 2x + 1}{x^2 - 3x + 2} = \frac{x^2 - 3x + 2 + 5x - 1}{(x-1)(x-2)} \quad \text{M1}$$
$$= 1 + \frac{5x-1}{(x-1)(x-2)} \quad \text{A1}$$

Let  $\frac{5x-1}{(x-1)(x-2)} = \frac{B}{x-1} + \frac{C}{x-2} = \frac{B(x-2) + C(x-1)}{(x-1)(x-2)}$  M1

Putting  $x = 1, 2$ ,  $B = -4, C = 9$  A1

OR

$$1 - \frac{4}{x-1} + \frac{9}{x-2} = \frac{(x-1)(x-2) - 4(x-2) + 9(x-1)}{(x-1)(x-2)} \quad \text{M1A1}$$
$$= \frac{x^2 - 3x + 2 - 4x + 8 + 9x - 9}{(x-1)(x-2)} \quad \text{M1}$$
$$= \frac{x^2 + 2x + 1}{(x-1)(x-2)} \quad \text{A1}$$
$$= f(x)$$

$$f'(x) = \frac{4}{(x-1)^2} - \frac{9}{(x-2)^2} \quad \text{B1B1}$$

$$f''(x) = -\frac{8}{(x-1)^3} + \frac{18}{(x-2)^3} \quad \text{B1}$$



2011

2. Gan ddefnyddio'r amnewid  $u = e^x$ , enrhifwch yr integrynn

$$\int_0^1 \frac{1}{(e^x + 4e^{-x})} dx.$$

Rhowch eich ateb yn gywir i dri lle degol.

[6]



2012

2

$$u = e^x \Rightarrow du = e^x dx,$$
$$[0,1] \rightarrow [1, e]$$

$$I = \int_1^e \frac{du}{u + 4/u}$$

$$= \int_1^e \frac{du}{u^2 + 4}$$

$$= \frac{1}{2} \left[ \tan^{-1} \left( \frac{u}{2} \right) \right]_1^e$$

$$= 0.236$$

B1

B1

M1

A1

A1

A1



2012

4. Mae'r ffwythiant  $f$  wedi'i roi gan

$$f(x) = \frac{3x^2 - 4x + 1}{(x - 2)(x^2 + 1)}.$$

- (a) Mynegwch  $f(x)$  yn nhermau ffracsynau rhannol. [4]
- (b) Trwy hyn, enrhwifwch

$$\int_3^4 f(x)dx,$$

gan roi eich ateb yn y ffurf  $\ln\left(\frac{a}{b}\right)$ , lle mae  $a, b$  yn gyfanrifau positif. [5]



2012

4(a) Let

$$\begin{aligned}\frac{3x^2 - 4x + 1}{(x-2)(x^2+1)} &\equiv \frac{A}{x-2} + \frac{Bx+C}{x^2+1} \\ &= \frac{A(x^2+1) + (Bx+C)(x-2)}{(x-2)(x^2+1)}\end{aligned}$$

M1

$$x = 2 \text{ gives } A = 1$$

A1

$$\text{Coeff of } x^2 \text{ gives } A + B = 3, B = 2$$

A1

$$\text{Const term gives } A - 2C = 1, C = 0$$

A1

(b)

$$\int_3^4 f(x) dx = \int_3^4 \frac{1}{x-2} dx + \int_3^4 \frac{2x}{x^2+1} dx$$

M1

$$= [\ln(x-2)]_3^4 + [\ln(x^2+1)]_3^4$$

A1A1

$$= \ln 2 - \ln 1 + \ln 17 - \ln 10$$

A1

$$= \ln\left(\frac{34}{10}\right) \text{ or } \ln\left(\frac{17}{5}\right)$$

A1



2012