MATHEMATEG FP2

Mathemateg Bur Bellach

Locysau mewn ffurfiau Cartesaidd a pharamedrig

5. Hafaliad yr elips E yw

$$16x^2 + 25y^2 = 400.$$

(a) Darganfyddwch gyfesurynnau ffocysau E. [4]

(b) Dangoswch fod y pwynt P â chyfesurynnau $(5\cos\theta, 4\sin\theta)$ ar E. [1]

(c) (i) Dangoswch y rhoddir hafaliad y normal i E yn P gan

$$4y\cos\theta - 5x\sin\theta + 9\sin\theta\cos\theta = 0.$$

(ii) Mae'r normal hwn yn croestorri'r echelin-x yn Q a'r echelin-y yn R. Dangoswch mai elips yw locws M, lle dynoda M ganolbwynt QR. [10]

5. *(a)* The equation in standard form is

$$\frac{x^2}{25}+\frac{y^2}{16}=1$$ M1

so $a = 5$ and $b = 4$. A1

The coordinates of the foci are $(\pm\sqrt{a^2-b^2},0)$ ie $(\pm3, 0)$ M1A1

(b) $\frac{(5\cos\theta)^2}{25}+\frac{(4\sin\theta)^2}{16}=1$ so the point lies on the ellipse. B1

(c) (i) $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y/\mathrm{d}\theta}{\mathrm{d}x/\mathrm{d}\theta}=-\frac{4\cos\theta}{5\sin\theta}$ M1A1

Gradient of normal = $\frac{5\sin\theta}{4\cos\theta}$ A1

Equation of normal is

$$y-4\sin\theta=\frac{5\sin\theta}{4\cos\theta}(x-5\cos\theta)$$ M1

$4y\cos\theta - 5x\sin\theta + 25\sin\theta\cos\theta - 16\sin\theta\cos\theta$ A1

Whence printed result.

(ii) Putting $y = 0$, cords of Q are $\left(\frac{9}{5}\cos\theta,0\right)$ B1

Putting $x = 0$, cords of R are $\left(0,-\frac{9}{4}\sin\theta\right)$ B1

Coords of M are $(x,y) = \left(\frac{9}{10}\cos\theta,-\frac{9}{8}\sin\theta\right)$ B1

Eliminating θ,

$$\frac{x^2}{(9/10)^2}+\frac{y^2}{(9/8)^2}=1\text{, ie an ellipse}$$ M1A1

[These 2 marks can be gained by noting that the above are the parametric equations of an ellipse]

5. *(a)* Show that the equation of the normal to the parabola $y^2 = 4ax$ at the point $P(ap^2, 2ap)$ is

$$y + px = ap(2 + p^2).$$ [4]

(b) This normal meets the x-axis at Q and the mid-point of PQ is R.

(i) Find the coordinates of R.

(ii) The locus of R as p varies is a parabola. Find the equation of this parabola and the coordinates of its focus. [8]

5	(a)	$\frac{dy}{dx} = \frac{dy/dp}{dx/dp} = \frac{2a}{2ap} = \frac{1}{p}$	M1A1
		Gradient of normal = $-p$	A1
		Equation of normal is	
		$y - 2ap = -p(x - ap^2)$	M1
		whence the printed result.	
	(b)(i)	Putting $y = 0$,	M1
		$x = a(2 + p^2)$	A1
		Coords of R are $(x,y) = \left(\frac{1}{2}(ap^2 + a\{2 + p^2\}), \frac{1}{2}(0 + 2ap)\right)$	M1
		$= \left(a(1 + p^2), ap\right)$	A1
	(ii)	Eliminating p,	
		$x - a = a \times \frac{y^2}{a^2}$ or $y^2 = a(x - a)$	M1A1
		Coordinates of focus = $(a + \frac{a}{4}, 0)$, ie $(\frac{5a}{4}, 0)$	M1A1

6. The ellipse E has equation

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad .$$

(a) Show that the equation of the tangent to E at the point $(a\cos\theta, b\sin\theta)$ is

$$bx\cos\theta + ay\sin\theta = ab. \qquad [5]$$

(b) This tangent meets the coordinate axes at P and Q, and the mid-point of PQ is R. Find the Cartesian equation of the locus of R as θ varies. [7]

6. (a) $$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = -\frac{b\cos\theta}{a\sin\theta}$$ M1A1

Equation of tangent is

$$y - b\sin\theta = -\frac{b\cos\theta}{a\sin\theta}(x - a\cos\theta)$$ M1A1

$$bx\cos\theta + ay\sin\theta = ab(\cos^2\theta + \sin^2\theta)$$ A1

whence the printed result.

(b) Putting $y = 0$, P is $\left(\frac{a}{\cos\theta}, 0\right)$ M1A1

Putting $x = 0$, Q is $\left(0, \frac{b}{\sin\theta}\right)$ A1

Therefore R is $\left(\frac{a}{2\cos\theta}, \frac{b}{2\sin\theta}\right)$ A1

Eliminating θ,

$$\cos\theta = \frac{a}{2x};\sin\theta = \frac{b}{2y}$$ M1

$$\frac{a^2}{4x^2} + \frac{b^2}{4y^2} = \cos^2\theta + \sin^2\theta = 1$$ M1A1

8. Hafaliad parabola yw

$$x^2 + 8y = 0.$$

(a) Darganfyddwch gyfesurynnau'r ffocws a hafaliad y cyfeirlin *(directrix)*. [3]

(b) (i) Dangoswch fod y pwynt $P\ (4p, -2p^2)$ ar y parabola ar gyfer pob gwerth o p.

(ii) Darganfyddwch hafaliad y tangiad i'r parabola yn y pwynt P.

(iii) O wybod bod y tangiad hwn yn mynd trwy'r pwynt $(\lambda, 2)$, dangoswch fod

$$2p^2 - \lambda p - 2 = 0\ .$$

Trwy hyn, dangoswch fod y ddau dangiad i'r parabola o unrhyw bwynt ar y llinell $y = 2$ yn berpendicwlar. [7]

8. (a) Writing the equation in the form

$$x^2 = -8y$$

we note that, in the usual notation, $a = 2$ and x,y are interchanged with the negative sign indicating that the graph is below the x-axis. M1

The focus is (0, – 2) and the directrix $y = 2$. A1A1

(b) (i) The result follows since

$$(4p)^2 + 8(-2p^2) = 0$$ B1

(ii) $\frac{dy}{dx} = \frac{-4p}{4} = -p$ B1

The equation of the tangent is

$y + 2p^2 = -p(x - 4p)$ M1

$y + px = 2p^2$ A1

(iii) This passes through $(\lambda,2)$ if

$2 + p\lambda = 2p^2$ M1

or $2p^2 - p\lambda - 2 = 0$

The 2 roots p_1, p_2 satisfy $p_1 p_2 = -1$ M1

And since the gradient of the tangent, m, satisfies $m = -p$, it follows that $m_1 m_2 = -1$ which is the condition for perpendicularity. A1

6. Hafaliad yr elips E yw

$$2x^2 + 3y^2 - 4x + 12y + 8 = 0.$$

Darganfyddwch

(a) cyfesurynnau canol E, [3]

(b) echreiddiad *(eccentricity)* E, [4]

(c) cyfesurynnau ffocysau E, [2]

(ch) hafaliadau cyfeirliniau *(directrices)* E. [2]

6.	(a)	The equation can be rewritten	
		$2(x-1)^2 + 3(y+2)^2 = 6$	M1A1
		The centre is (1, – 2)	A1
	(b)	The equation can be rewritten	
		$\frac{(x-1)^2}{3} + \frac{(y+2)^2}{2} = 1$	M1
		$a = \sqrt{3}, b = \sqrt{2}$	A1
		$1 - e^2 = \frac{2}{3}, \quad e = \frac{1}{\sqrt{3}}$	M1A1
	(c)	The foci are (0, – 2) ; (2, – 2)	B1B1
	(d)	The directrices are $x = -2$, $x = 4$.	B1B1

7. Hafaliad parabola yw

$$y^2 - 2y - 8x + 25 = 0.$$

(a) Darganfyddwch

(i) cyfesurynnau'r fertig,
(ii) cyfesurynnau'r ffocws,
(iii) hafaliad y cyfeirlin *(directrix)*. [6]

(b) Mae'r llinell $y = mx$ yn torri'r parabola yn y pwyntiau P_1 a P_2.

(i) Darganfyddwch hafaliad cwadratig sydd â chyfesurynnau-x y pwyntiau P_1 a P_2 yn wreiddiau iddo.
(ii) Trwy hyn, darganfyddwch raddiannau'r ddau dangiad o'r tarddbwynt i'r parabola. [7]

7(a)(i)	Completing the square, $(y-1)^2 = 8x - 24$ The vertex is therefore (3,1)	**M1** **A1** **A1**
(ii)	In the usual notation, $a = 2$ si The focus is (5,1)	**B1** **B1**
(iii)	The equation of the directrix is $x = 1$	**B1**
(b)(i)	Substituting $y = mx$, $m^2x^2 - 2mx - 8x + 25 = 0$	**M1** **A1**
(ii)	For coincident roots, $(2m+8)^2 = 100m^2$ $3m^2 - m - 2 = 0$ Solving using a valid method, $m = 1, -2/3$	**M1** **A1** **A1** **M1** **A1**