

# Fectorau

Differu

Pellter → Buanedd → Cyflymiad

Integru

Cyflymiad → Buanedd → Pellter

Paralel  $a = \lambda b$

Perpendicwlar  $a \cdot b = 0$

Maint fector [a] (Pythagoras)

(Yn posib i weithio allan hyd, buanedd a cyflymiad)

Pellter rhwng 2 fector  $(a - b)$

Pellter lleiaf rhwng 2 fector  $\frac{\delta(a - b)^2}{\delta t} = 0$



6. At time  $t$  s, a particle  $P$  has position vector  $\mathbf{r}$  m with respect to an origin  $O$  given by

$$\mathbf{r} = (2t - 5)\mathbf{i} + (t - 3)\mathbf{j} + (7 - 2t)\mathbf{k}.$$

- (a) Show that the distance of the particle from the origin at time  $t$  s is given by

$$OP^2 = 9t^2 - 54t + 83,$$

and find the value of  $t$  when  $P$  is closest to  $O$ . [5]

- (b) Find the velocity of  $P$  and determine its magnitude. [3]

- (c) Show that, when  $P$  is closest to  $O$ , the direction of the velocity of  $P$  is perpendicular to the line  $OP$ . [3]



6(a).  $\vec{OP} = \underline{r} = (2t-5)\underline{i} + (t-3)\underline{j} + (7-2t)\underline{k}$

$$OP^2 = (2t-5)^2 + (t-3)^2 + (7-2t)^2 \quad M1$$

$$= 4t^2 - 20t + 25$$

$$+ t^2 - 6t + 9$$

$$+ 4t^2 - 28t + 49$$

$$= 9t^2 - 54t + 83$$

convincing

A1

P is closest to O when  $OP^2$  is minimum M1

$$\frac{d}{dt}(OP^2) = 0 \quad \text{+ differentiation} \quad m1$$

$$18t - 54 = 0$$

$$t \approx \underline{\underline{3}}$$

A1



(b)  $\underline{r} = (-5\underline{i} - 3\underline{j} + \underline{k}) + (2\underline{i} + \underline{j} - 2\underline{k})t$  orthogonal MI

$\therefore \underline{v} = 2\underline{i} + \underline{j} - 2\underline{k}$  is constant AI

$$|\underline{v}| = \sqrt{2^2 + 1^2 + 2^2} = 3 \quad \text{AI}$$

(c) When P is closest to O,

$$\overrightarrow{OP} = \underline{i} + \underline{k} \quad \text{BI}$$

$$\overrightarrow{OP} \cdot \underline{v} = 2 \times 1 - 2 \times 1 \quad \text{MI}$$

$$= 0$$

$\therefore$  Direction of velocity of P is  $\perp$  to OP AI



6. A particle  $P$  moves such that its position vector  $\mathbf{r}$  with respect to the origin  $O$  at time  $t$  is given by

$$\mathbf{r} = \cos 3t\mathbf{i} + \sin 3t\mathbf{j}.$$

- (a) Find an expression for  $\mathbf{v}$ , the velocity of  $P$  at time  $t$ . [3]
- (b) Show that the direction of  $\mathbf{v}$  is perpendicular to that of  $\mathbf{r}$  for all values of  $t$ . [3]
- (c) Find the speed of  $P$ . [3]



6. (a)  $\mathbf{r} = \cos 3t \mathbf{i} + \sin 3t \mathbf{j}$

$$\mathbf{v} = \frac{d}{dt}(\mathbf{r}) \quad \text{used M1}$$

$$= -3 \sin 3t \mathbf{i} + 3 \cos 3t \mathbf{j} \quad \text{A2}$$

(b) Consider  $\mathbf{v} \cdot \mathbf{r} = (-3 \sin 3t \mathbf{i} + 3 \cos 3t \mathbf{j}) \cdot (\cos 3t \mathbf{i} + \sin 3t \mathbf{j}) \quad \text{M1}$

$$= -3 \sin 3t \cos 3t + 3 \sin 3t \cos 3t \quad \text{dot product B1}$$

$$= 0$$

$\therefore \mathbf{v}$  is perpendicular to  $\mathbf{r}$  for all values of  $t$  A1

(c) Speed of P =  $|\mathbf{v}|$  si M1

$$= \sqrt{(-3 \sin 3t)^2 + (3 \cos 3t)^2} \quad \text{M1}$$

$$= \sqrt{9(\sin^2 3t + \cos^2 3t)} \\ = 3 \quad \text{c.a.o. A1}$$



**5.** Rhoddir y fectorau  $\mathbf{a}$  a  $\mathbf{b}$  gan

$$\mathbf{a} = 2\mathbf{i} + 13\mathbf{j} - 10\mathbf{k},$$

$$\mathbf{b} = -\mathbf{i} + y\mathbf{j} + 5\mathbf{k}.$$

- (a) O wybod bod  $\mathbf{a}$  a  $\mathbf{b}$  yn berpendicwlar, darganfyddwch werth y. [4]
- (b) O wybod bod  $\mathbf{a}$  a  $\mathbf{b}$  yn baralel, darganfyddwch werth y. [2]



5. (a)  $\mathbf{a}, \mathbf{b}$  perpendicular  $\Rightarrow \mathbf{a} \cdot \mathbf{b} = 0$  M1  
$$\mathbf{a} \cdot \mathbf{b} = -2 + 13y - 50$$
 M1 A1  
$$-2 + 13y - 50 = 0$$
  
$$y = \underline{4}$$
 A1
- (b)  $\mathbf{a}, \mathbf{b}$  parallel  $\Rightarrow \mathbf{a} = -2\mathbf{b}$  M1  
$$-2y = 13$$
  
$$y = \underline{-6.5}$$
 A1



3. Mae gronyn, mäs 5 kg, yn symud mewn llinell syth dan effaith grym sengl. Rhoddir maint  $F$  N y grym hwn ar amser  $t$  s gan

$$F = 15t^2 - 60t, \quad t \geq 0.$$

- (a) Darganfyddwch gyflymiad y gronyn pan fydd  $t = 2$ . [2]
- (b) Dynodir cyflymder y gronyn ar amser  $t$  s gan  $v$  ms $^{-1}$ . O wybod bod  $v = 35$  pan fydd  $t = 0$ , darganfyddwch fynegiad ar gyfer  $v$  yn nhermau  $t$ . [4]
- (c) Cyfrifwch werth lleiaf buanedd y gronyn. [3]
- (ch) Darganfyddwch y pellter a deithir gan y gronyn rhwng  $t = 2$  a  $t = 8$ . [4]



3.(a) Using  $F = ma$        $5a = 15t^2 - 60t$       M1

$$a = 3t^2 - 12t$$

When  $t = 2$        $a = 12 - 24$   
                           $= -12$

Therefore magnitude of acceleration = -12 ms<sup>-1</sup>      A1

3.(b)       $v = \int 3t^2 - 12t dt$       M1

$$= t^3 - 6t^2 (+ C)$$
      A1

When  $t = 0, v = 35$       m1

$$C = 35$$
      A1

$$v = t^3 - 6t^2 + 35$$
      A1

3.(c) Least value of  $v$  when  $a = 0$       M1

$$3t(t - 4) = 0$$

$$t = (0 \text{ or}) 4$$
      ft  $v$       A1

Therefore least value of  $v = 4^3 - 6 \times 4^2 + 35$

$$= \underline{3 \text{ ms}^{-1}}$$
      ft  $v$       A1

3.(d) Required distance =  $\int_2^8 t^3 - 6t^2 + 35 dt$       attempt to integrate  $v$       M1

$$= \left[ \frac{t^4}{4} - 2t^3 + 35t \right]_2^8$$
      correct integration      A1

$$= (16 \times 64 - 16 \times 64 + 35 \times 8) - (4 - 16 + 70)$$
      m1

$$= 280 - 58$$
      cao      A1

$$= \underline{222 \text{ m}}$$
      cao      A1



1. Mae gronyn yn symud ar hyd yr echelin- $x$ . Ei gyflymder ar amser  $t$  s yw  $v \text{ ms}^{-1}$  ac mae

$$v = \cos 2t - 3\sin t.$$

- (a) Darganfyddwch gyflymiad y gronyn pan fydd  $t = \pi$ . [4]
- (b) O wybod bod  $x = 4$  pan fydd  $t = 0$ , cyfrifwch bellter y gronyn o'r tarddbwyt  $O$  pan fydd  $t = \frac{\pi}{4}$ . [6]



1.(a) 
$$a = \frac{d}{dt}(\cos 2t - 3 \sin 2t)$$

$= -2\sin 2t - 3\cos 2t$  A1 A1

When  $t = \pi$

$$a = -2\sin 2\pi - 3\cos \pi$$

$$a = 3 \text{ ms}^{-2}$$
 A1

1.(b)  $x = \int \cos 2t - 3 \sin t \, dt$  attempted M1

$$x = 0.5\sin 2t + 3\cos t + C$$
 A1 A1

When  $t = 0, x = 4$  used m1

$$4 = 3 + C$$

$$C = 1$$
 ft A1

$$x = 0.5\sin 2t + 3\cos t + 1$$

$$x = \underline{3.62 \text{ m}}$$
 cao A1



2. Fector safle gronyn  $P$  ar amser  $t$  s yw  $\mathbf{r}$  m ac mae

$$\mathbf{r} = (3t^2 + 1)\mathbf{i} + (13t - 2t^2)\mathbf{j}.$$

- (a) Darganfyddwch fuanedd  $P$  pan fydd  $t = 2$ . [4]
- (b) Cyfrifwch werth  $t$  pan fydd cyflymder  $P$  yn berpendicwlar i'r fector  $2\mathbf{i} - \mathbf{j}$ . [3]
- (c) Dangoswch fod cyflymiad  $P$  yn gyson a darganfyddwch ei faint. [3]
- (ch) Darganfyddwch yr ongl rhwng cyfeiriad cyflymiad  $P$  a chyfeiriad cyflymder  $P$  pan fydd  $t = 2$ . [3]



2. (a)  $\mathbf{v} = \frac{d\mathbf{r}}{dt}$  usedM1  
 $\mathbf{v} = 6t\mathbf{i} + (13 - 4t)\mathbf{j}$  A1

Speed =  $\sqrt{(6t)^2 + (13 - 4t)^2}$  M1

When  $t = 2$ , speed =  $\sqrt{144 + 25} = \underline{13}$  ft A1

(b) velocity is perpendicular to  $(2\mathbf{i} - \mathbf{j})$  when  $\mathbf{v} \cdot (2\mathbf{i} - \mathbf{j}) = 0$  M1

$12t - 13 + 4t = 0$  method for dot product M1

$t = \frac{13}{16}$  cao A1

(c) Acceleration of  $P = \mathbf{a} = \frac{d\mathbf{v}}{dt}$  usedM1  
 $\mathbf{a} = 6\mathbf{i} - 4\mathbf{j}$  independent of  $t$  ft v A1

Magnitude =  $\sqrt{36 + 16} = \sqrt{52}$  ft A1

(d) Let  $\theta$  be the required angle.  
Use of  $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos\theta$  with  $\mathbf{b} = \mathbf{v}$  when  $t = 2$  M1

$(6\mathbf{i} - 4\mathbf{j}) \cdot (12\mathbf{i} + 5\mathbf{j}) = \sqrt{52} \times 13 \cos\theta$  ft A1

$72 - 20 = \sqrt{52} \times 13 \cos\theta$

$\theta = \underline{56.3^\circ}$  cao A1



HAF 2010

1. Mae gronyn yn symud ar hyd yr echelin- $x$ . Ei gyflymder ar amser  $t$  s yw  $\text{vms}^{-1}$  ac mae

$$v = 12 \sin 3t - 8 \cos 2t.$$

- (a) Darganfyddwch fynegiad ar gyfer cyflymiad y gronyn ar amser  $t$  s. [3]
- (b) O wybod bod y gronyn yn y tarddbwynt  $O$  ar amser  $t = 0$ , darganfyddwch fynegiad ar gyfer dadleoliad y gronyn o  $O$  ar amser  $t$  s. [5]



1(a)

$$a = \frac{dv}{dt}$$

$$a = 36\cos 3t + 16\sin 2t$$

M1

A1A1

1(b)

$$x = \int 12\sin 3t - 8\cos 2t \, dt$$

$$x = -4\cos 3t - 4\sin 2t + (C)$$

$$t = 0, x = 0$$

$$0 = -4 + C$$

$$C = 4$$

M1

A1A1

m1

A1


**HAF 2011**