Tensiynau ergydiol Gronynnau cysylltiedig

M3

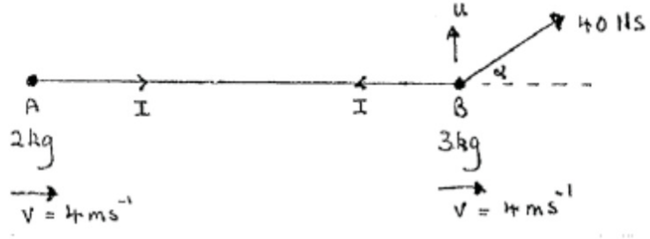
5. Particle A, of mass 2 kg, and particle B, of mass 3kg, are connected by a light inextensible string of length l m. Initially, both particles are lying at rest on a smooth horizontal surface a distance l m apart, with the string just slack. Particle B is given a blow of impulse 40 Ns in a direction away from A at an angle α to the line joining the initial positions of A and B.

Immediately after the blow, the speed of particle A is 4 ms^{-1}.

(a) Determine the value of α. [6]

(b) Calculate the magnitude and direction of the velocity of B immediately after the blow. [6]

5. *(a)*



Impulse = change in momentum

Apply to A $\quad \mathrm{I} = 2v$ — M1

$\quad = 2 \times 4$ — A1

Apply to B $\quad -\mathrm{I} = -40\cos\alpha + 3v$ — M1

$\quad = -40\cos\alpha + 3 \times 4$ — A1

$\therefore \quad -8 = -40\cos\alpha + 12$ — M1

$40\cos\alpha = 20$

$\cos\alpha = \frac{1}{2}$

$\alpha = \underline{60^\circ}$ — A1

(b) $\quad 40\sin\alpha = 3u$

$u = 40 \times \frac{\sqrt{3}}{2} \times \frac{1}{3}$

$\quad = \frac{20\sqrt{3}}{3}$ — A1

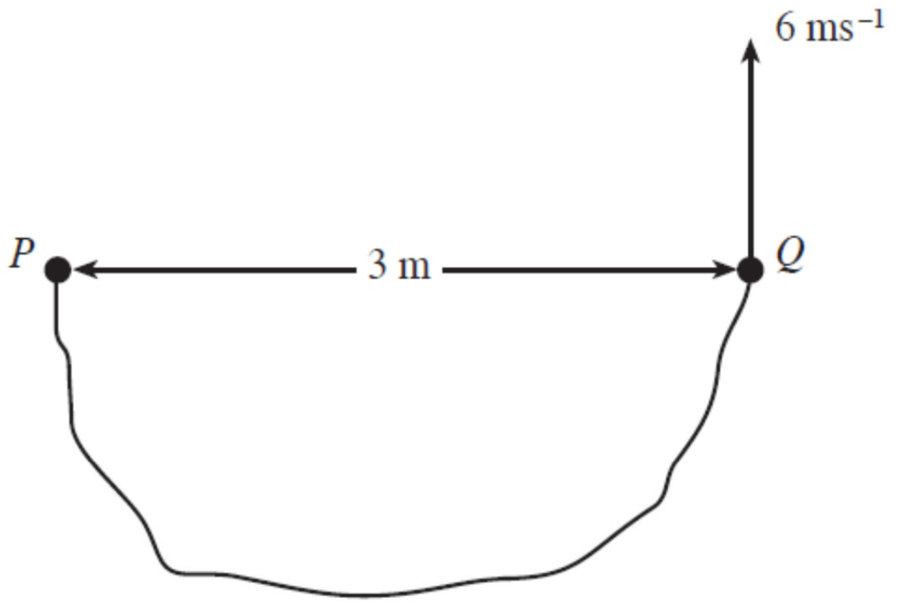
Speed of b $\quad = \sqrt{\left(\frac{20\sqrt{3}}{3}\right)^2 + 4^2}$ — M1

$\quad = \underline{12{\cdot}22\ \mathrm{ms}^{-1}}$ — A1

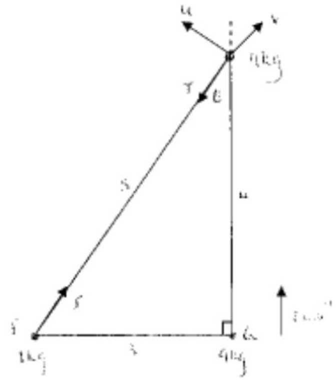
$\theta = \tan^{-1}\left(\frac{20\sqrt{3}}{3 \times 4}\right)$ — M1

$\quad = \underline{70{\cdot}89^\circ}$ — A1

4. Two particles P and Q, of mass 7 kg and 9 kg respectively, are attached one to each end of a light inextensible string of length 5 m. Initially, the particles are at rest on a smooth horizontal surface a distance 3 m apart, as shown in the diagram. Particle Q is then projected horizontally with velocity $6\,\text{ms}^{-1}$ in a direction at 90° to the line joining the initial positions of P and Q.



Calculate the speed of P and the speed of Q immediately after the string becomes taut. Determine the impulsive tension in the string during the jerk, and find the angle between the velocity of P and the velocity of Q immediately after the jerk. [14]

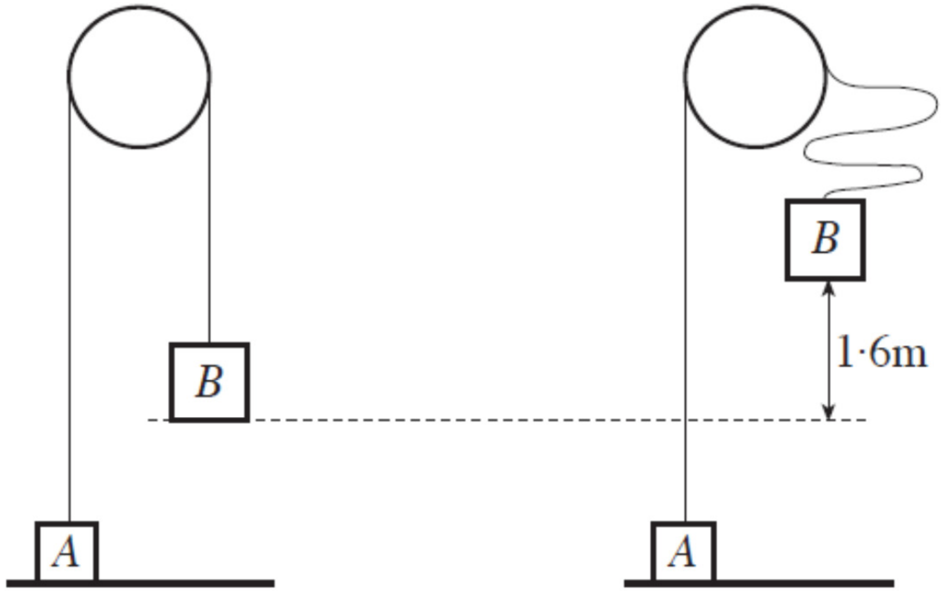


$\cos\theta = 0.8$,	$\sin\theta = 0.6$	si	B1
$u = 6\sin\theta$			M1
$u = 6 \times 0.6 = 3.6 \text{ ms}^{-1}$		si	A1

Impulse = change in momentum

For P		M1
$J = 7v$		A1
For Q		M1
$9 \times 6\cos\theta - J = 9v$		A1
Adding $54 \times 0.8 = 16v$		m1
$v = 2.7 \text{ ms}^{-1}$		A1
Speed of P = $\underline{2.7 \text{ ms}^{-1}}$		
Speed of Q = $\sqrt{2.7^2 + 3.6^2}$		M1
= $\underline{4.5 \text{ ms}^{-1}}$	ft 2.7(c)	A1
$J = 7 \times 2.7$		
= $\underline{18.9 \text{ Ns}}$	ft 2.7(c)	A1
Required angle $\alpha = \tan^{-1}\left(\dfrac{u}{v}\right)$		M1
$= \tan^{-1}\left(\dfrac{3.6}{2.7}\right)$		
$= \underline{53.13^{\circ}}$	ft	A1

6. A particle A, of mass 7 kg, rests on a horizontal table. It is attached to one end of a light inextensible string which passes over a smooth light pulley. The other end of the string is attached to another particle B, of mass 3 kg. Initially, the particles are held at rest with the string just taut. Particle B is raised vertically through a distance of 1·6 m and released from rest.



Find the speed with which particle A begins to rise, and the impulsive tension in the string. [9]

6.	Using $v^2 = u^2 + 2as$ with $u = 0$, $a = (-)9.8$, $s = 0$		M1
	$v^2 = 2 \times 9.8 \times 1.6$		A1
	$v = 5.6 \text{ ms}^{-1}$		A1
	Impulse = change in momentum	applied to both particles	M1
	For *A* $J = 7v'$		B1
	For *B* $J = 3 \times 5.6 - 3v'$	ft v	A1
	Solving $7v' = 16.8 - 3v'$		m1
	$10v' = 16.8$		
	$v' = 1.68$	ft v	A1
	$J = 11.76$ Ns	ft v	A1

4. Two spheres P and Q, of mass 5 kg and 3 kg respectively, rest on a smooth table. They are connected by a light inextensible string which is initially slack. An impulse of magnitude 1·2 Ns is applied to Q in the direction PQ.

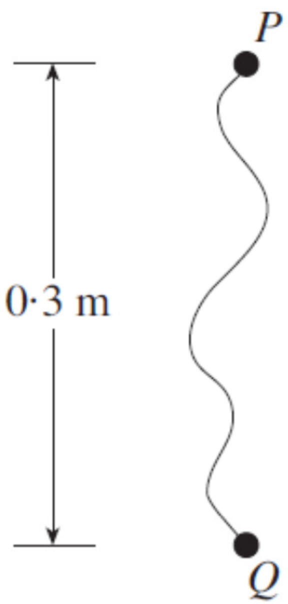
(a) Determine the speed with which Q begins to move. [2]

(b) Find the speed with which P moves after the string tightens, and determine the impulsive tension in the string. [6]

(c) Calculate the loss in energy when the string tightens. [4]

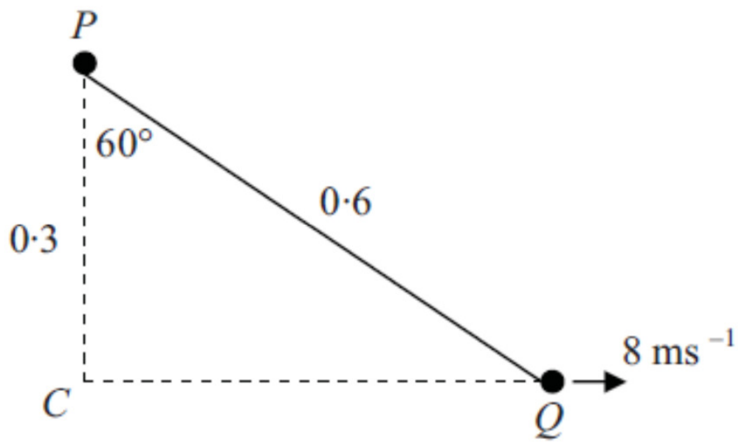
4.(a)	Impulse = change in momentum	used	M1
	$1.2 = 3v$		
	$v = \underline{0.4 \text{ ms}^{-1}}$	cao	A1
4.(b)	For Q $\quad -I = 3v - 3 \times 0.4$	attempt P or Q	M1
	$I = 3v - 1.2$		
	For P $\quad I = 5v$	attempt	m1
	Both equations correct		A1
	Solving simultaneously		m1
	$5v = 1.2 - 3v$		
	$8v = 1.2$		
	$v = \underline{0.15 \text{ ms}^{-1}}$	cao	A1
	$I = \underline{0.75 \text{ Ns}}$	cao	A1
4.(c)	Loss in energy $= 0.5 \times 3 \times 0.4^2 - 0.5 \times 8 \times 0.15^2$	ft v's	M1 A1 A1
	$= \underline{0.15 \text{ J}}$	cao	A1

4. Mae dau ronyn *P* a *Q*, masau 3 kg a 5 kg yn ôl eu trefn, ynghlwm wrth naill ben a llall llinyn ysgafn anestynadwy, hyd 0·6 m. I ddechrau, mae'r gronynnau'n ddisymud ar arwyneb llorweddol llyfn, bellter 0·3 m oddi wrth ei gilydd, fel yn y diagram.

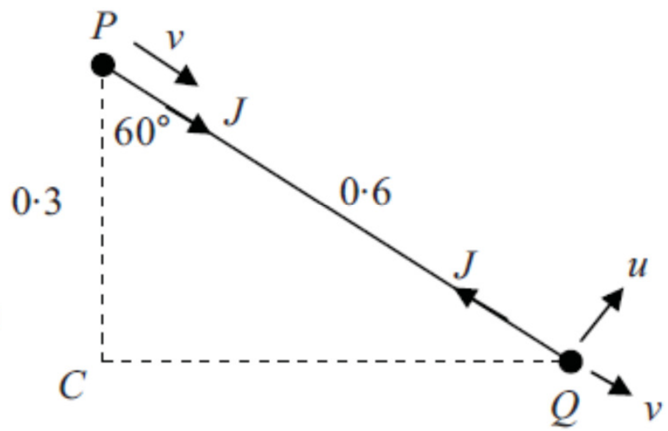


Mae'r gronyn *Q* yn cael ei daflu ar draws yr arwyneb â buanedd 8 ms^{-1} mewn cyfeiriad sydd ar ongl 90° i'r llinell sy'n cysylltu safleoedd cychwynnol *P* a *Q*. Darganfyddwch y tensiwn ergydiol yn y llinyn yn ystod y plwc, gan nodi eich unedau'n glir. Darganfyddwch fuanedd y naill ronyn a'r llall wrth iddynt ddechrau symud yn syth ar ôl y plwc. [11]

4. When the string jerks tight, each particle begins to move in direction PQ with equal speeds v.



Just before jerk



Just after jerk

$$\cos \angle CPQ = \frac{1}{2}$$

$$\sin \angle CPQ = \frac{\sqrt{3}}{2}$$ si B1

Use of impulse = change in momentum M1

Applied to P $J = 3v$ B1

Applied to Q $J = 5 \times 8\sin 60° - 5v$ A1

Attempt to solve simultaneously m1

$$3v = 40 \times \frac{\sqrt{3}}{2} - 5v$$

$$v = \frac{5\sqrt{3}}{2} = \underline{4.33\ (\text{ms}^{-1})}$$ cao A1

Speed of particle P is 4.33 ms^{-1}.

Magnitude of impulsive tension $= J = 3v$

$$= \frac{15\sqrt{3}}{2} = \underline{12.99\ (\text{Ns})}$$ cao A1

units B1

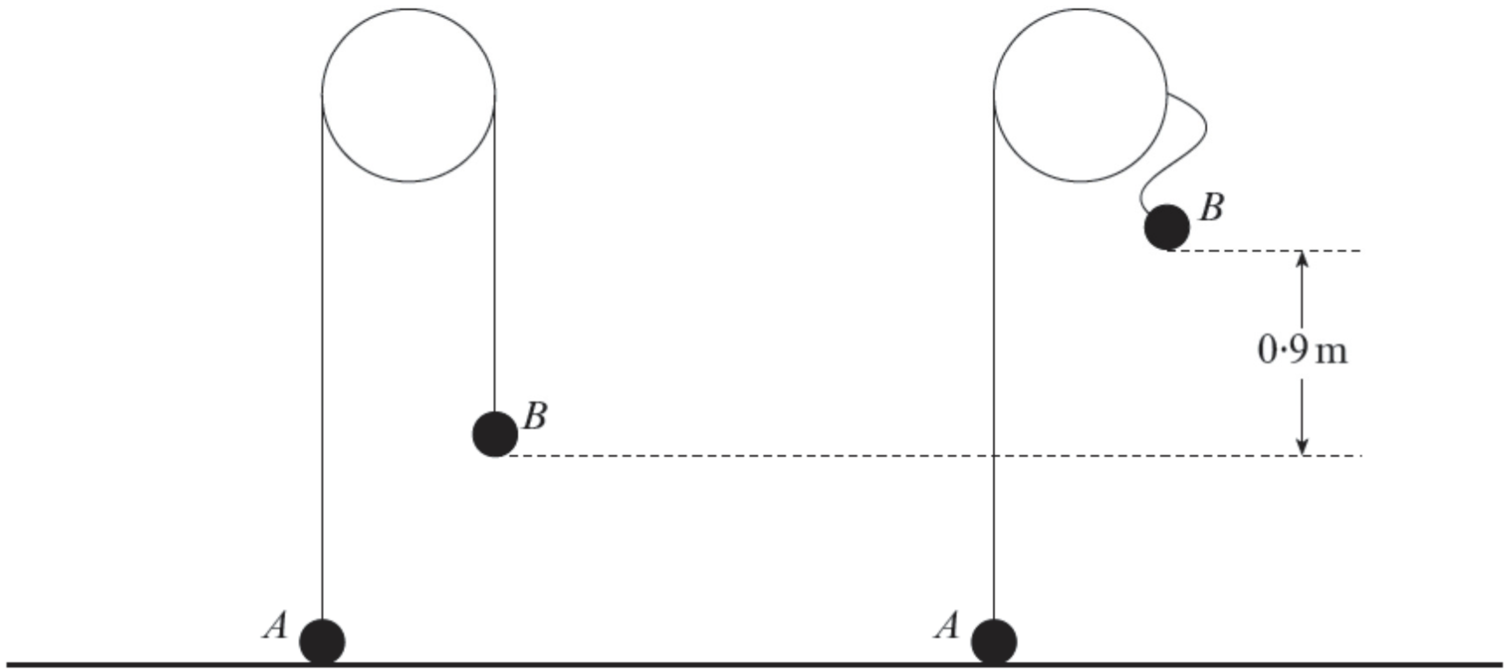
Perpendicular to PQ, there is no impulse

Speed of particle Q perpendicular to PQ $= 8\cos 60° = 4\ \text{ms}^{-1}$ B1

Speed of particle Q $= \sqrt{4^2 + \left(\frac{5\sqrt{3}}{2}\right)^2}$ M1

$$= \underline{5.89\ \text{ms}^{-1}}$$ cao A1

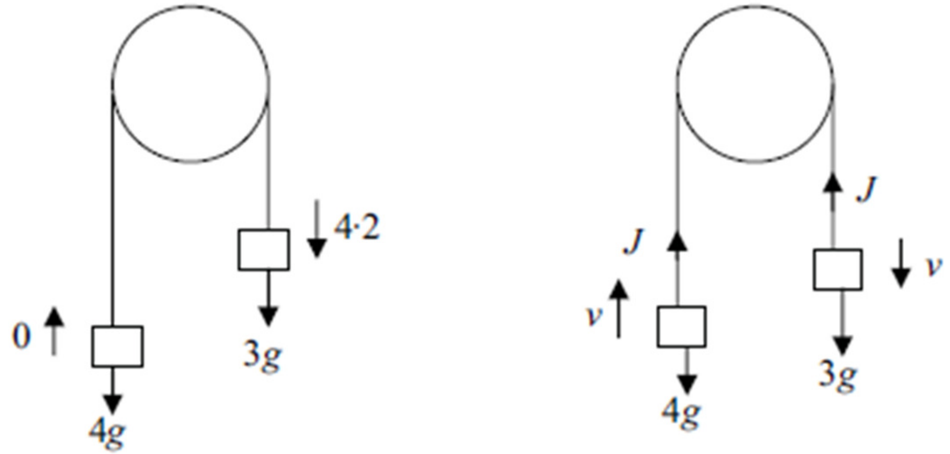
5. Mae'r diagram yn dangos dau ronyn, *A* a *B*, masau 4 kg a 3 kg yn ôl eu trefn, wedi'u cysylltu â'i gilydd gan linyn ysgafn anestynadwy yn mynd dros bwli ysgafn llyfn sy'n sefydlog uwchben plân llorweddol. I ddechrau, mae'r gronyn *A* yn ddisymud ar y plân ac mae'r gronyn *B* yn hongian ar ddyfnder 1·0 m islaw y pwli.



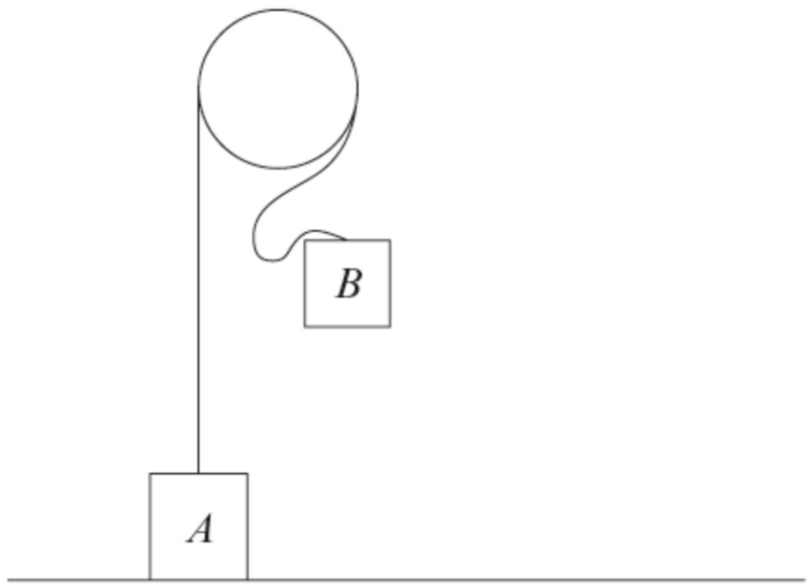
Yna, caiff gronyn *B* ei godi'n fertigol trwy bellter 0·9 m a'i ryddhau o ddisymudedd o'r safle hwn.

(a) Cyfrifwch fuanedd *B* yn union cyn i'r llinyn dynhau. [3]

(b) Darganfyddwch fuanedd *A* wrth iddo adael y plân a'r tensiwn ergydiol yn y llinyn yn syth ar ôl i'r llinyn dynhau. [7]

5(a).	Using $v^2 = u^2 + 2as$ with $u = 0$, $a = 9.8$, $s = 0.9$ (downwards positive)	M1
	$v^2 = 0 + 2 \times 9.8 \times 0.9$	A1
	$v = \underline{4.2\ (\text{ms}^{-1})}$	A1
5(b)	Before After	
		
	$J = 3(4.2 - v)$	M1A1
	$J = 4v$	M1A1
	$12.6 - 3v = 4v$	m1
	$7v = 12.6$	
	$v = \underline{1.8\ \text{ms}^{-1}}$	A1
	$J = 4v$	
	$J = \underline{7.2\ (\text{Ns})}$	A1

5. Mae gronyn A, màs $5\,\text{kg}$, yn ddisymud ar arwyneb llorweddol. Mae ynghlwm wrth un pen llinyn ysgafn anestynadwy sy'n mynd dros bwli ysgafn llyfn sefydlog. Mae pen arall y llinyn ynghlwm wrth ronyn arall B, màs $2\,\text{kg}$. I ddechrau, mae'r gronynnau wedi'u cynnal yn ddisymud gyda'r llinyn prin yn dynn. Yna, caiff gronyn B ei godi'n fertigol a'i ryddhau o ddisymudedd. Ar ôl iddo syrthio am $0{\cdot}5\,\text{s}$, mae'r llinyn yn tynhau.



Darganfyddwch fuanedd y gronyn A wrth iddo ddechrau codi a'r tensiwn ergydiol yn y llinyn. [8]

5.	Using $v = u + at$ with $u=0$, $a=(\pm)9.8$, $t=2.5$	M1
	$v = 9.8 \times 0.5$	
	$v = 4.9\ ms^{-1}$	A1
	Impulse = Change in momentum	M1
	For A $J = 5v$	B1
	For B $J = 2 \times 4.9 - 2v$	A1
	Solving $5v = 9.8 - 2v$	m1
	$7v = 9.8$	
	$v = \underline{1.4\ (ms^{-1})}$	A1
	$J = 5 \times 1.4$	
	$J = \underline{7\ (Ns)}$	A1