A Level Mathematics C3
January 2009
Marking Scheme

1. $h = \dfrac{2\pi}{\frac{9}{4}} = \dfrac{\pi}{18}$ — M1 (correct formula with $h = \pi/18$)

Integral $= \dfrac{\pi}{3 \times 18}$ [0 + (–0.26651509) + 4 (–0.01530883 – 0.14384104)
+ 2(–0.06220246)] — B1 (3 values)
B1 (other 2 values)

≈ -0.0598 — A1 (F.T. one slip)

$\int_0^{\frac{2\pi}{9}} \ln(\cos^2 x)\,dx \approx 2(-0.0598) = -0.1196$ — B1

(5)

2. (a) $\theta = 0$, $\cos 2\theta = 1$, for example — B1 (choice of θ and one correct evaluation)
$2\cos^2\theta - \sin^2\theta = 2$ — B1

(statement is false)

(b) $3(\sec^2\theta - 1) = 7 + \sec\theta$ — M1 (use of correct formula)
$3\sec^2\theta - \sec\theta - 10 = 0$ — M1 (attempt to solve quadratic, or correct formula or
$(3\sec\theta + 5)(\sec\theta - 2) = 0$ — $(a\sec\theta + b)(c\sec\theta + d)$ with $ac = 3$ $bd = -10$)

$\sec\theta = -\dfrac{5}{3}, 2$

$\cos\theta = -\dfrac{3}{5}, \dfrac{1}{2}$ — A1 (values of $\cos\theta$)
$\theta = 126.9^\circ, 233.1^\circ, \quad 60^\circ, 300^\circ$ — B1 (126.9°) B1 (233.1°)
(allow to nearest degree) — B1 (60°, 300°)

(8)

3. (a) $2x + 3x\dfrac{dy}{dx} + 3y + 4y\dfrac{dy}{dx} - 2 = 0$ — B1 $\left(3x\dfrac{dy}{dx} + 3y\right)$ (o.e)

$2 + 3\dfrac{dy}{dx} + 6 + 8\dfrac{dy}{dx} - 2 = 0$ — B1 $\left(4y\dfrac{dy}{dx}\right)$ (o.e)

B1 (correct diff ^{n} of $x^2, -2x$ and 13)

$\dfrac{dy}{dx} = -\dfrac{6}{11}$ — B1 (F.T. one slip)

(b) $\dfrac{dy}{dx} = \dfrac{8e^{2t} + 3e^{t}}{2e^{t}}$ — M1

B1 (numerator $ke^{2t} + 3e^{t}, \; k = 4, 8$)

B1 (k = 8)
B1 (denominator)

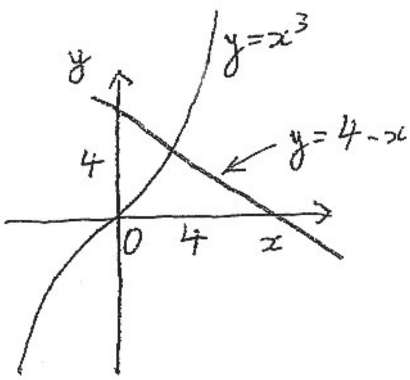
$\dfrac{8e^{2t} + 3e^{t}}{2e^{t}} = 6$ — M1

$8e^{t} = 9$ — M1

$t = ln\left(\dfrac{9}{8}\right) \approx 0.118$ — A1 (C.A.O)

(11)

4. (a)



B1 ($y = x^3$)
B1 ($y = 4 - x$)

B1 one real root
∵ one intersection

(b) $x_0 = 1.4,\ x_1 = 1.37506.....,\ x_2 = 1.37945........$ — B1 (x_1)

$x_3 = 1.37868....,\ x_4 = 1.37881.... \approx 1.3788$ — B1 (x_4 4 places)

Check 1.37875, 1.375885

M1 (attempt to find signs or values)

x	$f(x)$
1.37875	− 0.00031
1.37885	·0.0036

A1 (correct)

Changes of sign indicates presence of root which is 1.3788 correct to 4 dec. places — A1 (conclusion)

(8)

5. (a) (i) $\dfrac{1}{sin\ x}$ x $cos\, x$ M1 $\left(\dfrac{f\,(x)}{sin\, x},\ f\,(x) = \pm\ cos\ x\right)$

$= cot\ x$ A1 $(f\,(x) = cos\ x)$ A1 *(cot x)*

$\left(accept\ \dfrac{1}{tan\ x}\right)$

(ii) $\dfrac{4}{\sqrt{1-(4x)^2}}$ (o.e.) M1 $\dfrac{k}{\sqrt{1-(4x)^2}}$

A1 $(k = 4)$

(iii) $\dfrac{(x^2+5)(6x)-(3x^2+2)(2x)}{(x^2+5)^2}$ M1 $\left(\dfrac{(x^2+5)\,f\,(x)-(3x^2+2)\,g(x)}{(x^2+5)^2}\right)$

A1 $(f(x) = 6x,\ g(x) = 2x)$

$= \dfrac{26x}{(x^2+5)^2}$ A1

(b) $x = \tan y$

$1 = \sec\ y^2\ \dfrac{dy}{dx}$ M1 $(l = f(y)\ \dfrac{dy}{dx},\ f\,(y) \neq k)$

A1 $(f\,(y) = \sec^2\ y)$

$\dfrac{dy}{dx} = \dfrac{1}{\sec^2\ y}$

$= \dfrac{1}{1+\tan^2\ y}$ A1

$= \dfrac{1}{1+x^2}$ A1 (C.A.O)

(12)

6. (a) $2|x| + 9 = 5|x| + 5$

$3|x| = 4$ — B1 $\begin{pmatrix} a|x| = b \\ a = 3,\ b = 4 \end{pmatrix}$

$x = \pm 4/3$ — B1 (both values) (F.T. *a, b*)

(b) $5x + 7 \le -4,\ x \le -\frac{3}{5}$ — B1

and $5x + 7 \ge -4$ — M1

$x \ge -\frac{11}{5}$

$-\frac{11}{5} \le x \le -\frac{3}{5}$ — A1

(5)

7. (a) (i) $\frac{7}{6}\ln|6x+5| + c$ — M1 $(k\,ln|6x+5|, k = 7, \frac{7}{6})$

A1 $\left(k = \frac{7}{6}\right)$

(ii) $\frac{1}{5}\sin 5x + c$ — M1 $\left(k \sin 5x,\ k = \pm\frac{1}{5}, 5, 1\right)$

A1 $\left(k = \frac{1}{5}\right)$

(b) $\left[-\frac{9}{2(2x+1)}\right]_0^1$ — M1 $\left(\frac{k}{2x+1},\ k = -9, \pm\frac{9}{2}\right)$

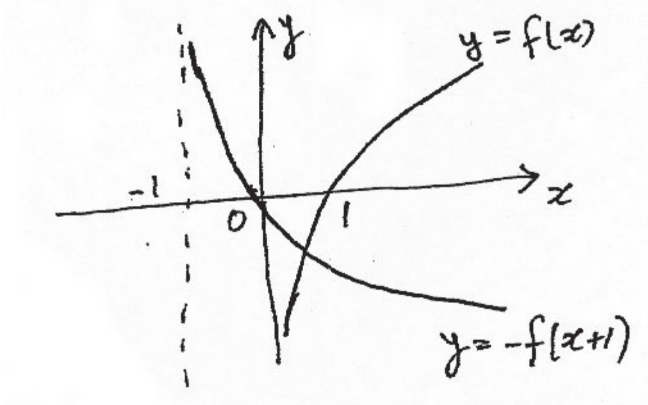
A1 $\left(k = -\frac{9}{2}\right)$

$= -\frac{9}{2}\left[\frac{1}{3} - 1\right]$ — M1 $\left(k\left(\frac{1}{3} - 1\right)\right)$ allowable *k*

$= 3$ — A1 $\left(\text{allow F.T for } k = \pm\frac{9}{2}\right)$

(8)

8.



$y = f(x)$	B1 (asymptote and shape)
	B1 (1,0)
$y = -f(x+1)$	B1 (asymptote)
	B1 (0,0)
	B1 (behaviour, large y)

(5)

9. (a) Let $y = 5x^2 + 4$

$y - 4 = 5x^2$ — M1 $(y-4=5x^2)$

$x = \pm\sqrt{\dfrac{y-4}{5}}$ — A2 $(\pm)$ OR A1 $(+)$ A1 $\left(\pm\dfrac{\sqrt{y-4}}{5}\right)$

$x = -\sqrt{\dfrac{y-4}{5}}$ — since domain $x \le 0$ — A1

$f^{-1}(x) = -\sqrt{\dfrac{x-4}{5}}$ — (F.T $x = f(y)$) — A1

(b) domain $x \ge 4$, Range $x \le 0$ (o.e) — B1

(6)

10. (a) Range of $f(x) \ge 2 - k$ (o.e) — B1

(b) $2 - k \ge 0$ — B1

$k \le 2$ — B1

(Greatest value of k is 2)

(c) $3(4-k)^2 + 4 = 31$ — M1 (attempt to form equation, correct order, un......)

$(4-k)^2 = 9$ — A1

$k = 1, 7$ — A1

$\therefore k = 1$ — A1 (F.T max value of k from (b))

(since $k \le 2$)

(7)

Mathematics FP1 January 2009

Solutions and Mark Scheme

Final Version

1	(a)	$\ln y = x\ln 2$	B1
		$\frac{1}{y}\frac{dy}{dx} = \ln 2$	B1
		$\frac{dy}{dx} = y\ln 2 = 2^x \ln 2$	B1
	(b)	$f(x+h) - f(x) = \frac{x+h}{x+h+1} - \frac{x}{x+1}$	M1
		$= \frac{(x+1)(x+h) - x(x+h+1)}{(x+h+1)(x+1)}$	A1
		$= \frac{x^2 + x + hx + h - x^2 - hx - x}{(x+h+1)(x+1)}$	A1
		$= \frac{h}{(x+h+1)(x+1)}$	A1
		$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$	
		$= \lim_{h \to 0} \frac{h}{h(x+h+1)(x+1)}$	M1
		$= \frac{1}{(x+1)^2}$	A1
2		$S_n = \sum_{r=1}^{n}(2r-1)^2$	M1
		$= 4\sum_{r=1}^{n} r^2 - 4\sum_{r=1}^{n} r + \sum_{r=1}^{n} 1$	A1
		$= \frac{4n(n+1)(2n+1)}{6} - \frac{4n(n+1)}{2} + n$	A1A1A1
		$= \frac{n}{3}\left[4n^2 + 6n + 2 - 6n - 6 + 3\right]$	
		$= \frac{n(4n^2 - 1)}{3}$	
		$= \frac{n(2n+1)(2n-1)}{3}$ cao	A1