Mathematics C3

1.		$h = 0{\cdot}2$	M1 (Correct formula h = 0.2)
		Integral $\approx \frac{0\cdot 2}{3}$ [1 + 1·8964809 + 4 (1·0408108	
		+ 1·4333294)	B1 (3 correct values)
		+ 2 (1·1735109)]	B1 (2 correct values)
		$\approx 1{\cdot}0093$	A1 (F.T. one slip)

(4)

2.	*(a)*	$\theta = \frac{\pi^c}{2}$ or degrees	B1 (choice of θ and use correct evaluation)
		$\sin 3\theta = \sin \frac{3\pi}{2} = -1$	B1 (2 correct evaluations)
		$4 \sin \frac{\pi}{2} - 3 \sin^3 \frac{\pi}{2} = +1$	
		$(\therefore \sin 3\theta \neq 4 \sin\theta - 3\sin^3\theta)$	
	(b)	see θ = $1 - 2(\sec^2\theta - 1)$	M1 ($\tan^2\theta = \sec^2\theta - 1$)
		$2\sec^2\theta + \sec\theta - 3 = 0$	M1((a sec θ +b)(c sec θ + d)
		$(2\sec\theta + 3)(\sec\theta - 1) = 0$	with ac = coefft of $\sec^2\theta$
		$\sec\theta = -\frac{3}{2}$, $\sec\theta = 1$	bd = constant term or use of correct formula)
		$\cos\theta = -\frac{3}{2}$, $\cos\theta = 1$	A1
		$\theta = 131{\cdot}8^\circ, 228{\cdot}2^\circ, 0^\circ, 360^\circ$	B1 (131·8°) B1(228·2°) B1 (0°, 360°)

(8)

3. *(a)* $\frac{dy}{dx} = \frac{2e^{2t}}{4t^3}$

M1 ($\dot{y} = ke^{2t}$, k = 1 or 2 or $2e^{2t} + 5$)

A1 ($2e^{2t}$)

M1 $\left(\frac{dy}{dx} = \frac{\dot{y}}{\dot{x}}\right)$, A1 (all correct C.A.O.)

(b) $4x^3 + \cos y \frac{dy}{dx} + 2xy^3 + 3x^2y^2 \frac{dy}{dx} = 0$

B1 (cos $y\frac{dy}{dx}$)

B1 ($2xy^3 + 3x^2 \frac{dy}{dx}$)

$$\frac{dy}{dx} = -\frac{4x^3 + 2xy^3}{\cos y + 3x^2y^2}$$

B1 $\left(\text{F.T } \frac{d}{dy}(\sin y) = -\cos y\frac{dy}{dx} \right)$

(7)

4.

x	$2\ln(x+70) - x$
8	0.71
9	−0·26

M1 (attempt to find values)

Change of sign indicates α is between 8 and 9

A1 (correct values or signs and conclusion)

x_0 = 8·8, x_1 = 8·7338 ….. , x_2 = 8·7321 ..

$x_3 \approx$ 8·7321

B1 (x_1)

B1 (x_3)

Check 8·73205, 8·73215

x	$f(x)$
8.73205	0.00005
8.73215	−0.0005

M1 (attempt to find relevant values or signs)

A1 (correct values)

Change of sign indicates α is 8·7321 correct to four decimal places

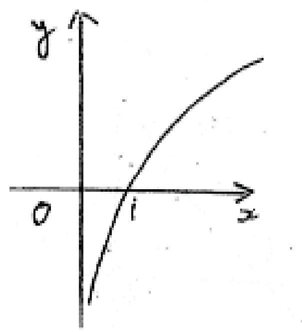
A1 (FT for incorrect values)

(7)

5. *(a)*	$\dfrac{x^2\,\frac{1}{x} - (\ln x)(2x)}{x^4} = \dfrac{1-2\ln x}{x^3}$	M1 $\dfrac{(x^2)(f(x) - (\ln x)(g(x))}{x^4}$
		A1 ($f(x) = \dfrac{1}{x}, g(x) = 2x$)
		A1 (simplified answer)
(b)	$\dfrac{-5}{\sqrt{1-25x^2}}$ (o.e)	M1 $\left(\dfrac{-k}{\sqrt{1-(5x)^2}} \right)$ k = 1, $\pm$ 5)
	$\left(\text{B1 for } \dfrac{-5}{\sqrt{1 - 5x^2}} \right)$	A1 $k = 5$
(c)	$\dfrac{1}{2}\left(1+6x^4\right)^{-\frac{1}{2}} 24x^3$	M1 ($\dfrac{1}{2}(1+6x^4)^{-\frac{1}{2}} f(x)$
		$f(x) = 24x^n, n = 1,2,3$
		$f(x) = kx^3$)
	$= 12x^3(1+6x^4)^{-\frac{1}{2}}$	A1 ($f(x) = 24x^3$, unambiguous simplified statement)
(d)	$2x^3\sec^2 2x + 3x^2\tan 2x$	M1 ($x^3 f(x) + \tan 2x g(x)$)
		A1 ($f(x) = k\sec^2 2x, k = 1,2$)
		A1 ($k = 2$, unambiguous answer)
		(10)

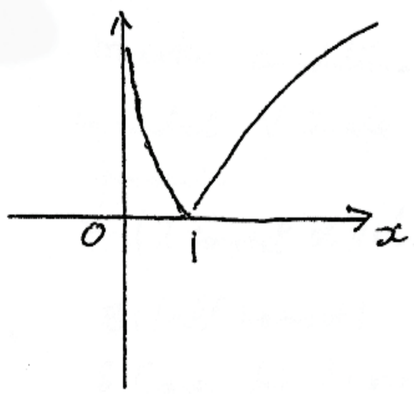
6.

(a) (i)



(i) M1 (shape)
A1((1,0), all correct)

(ii)



M1 (graph above x −axis and former shape for x >1)
A1 (x <1 correct)
F.T −ve parts of graph from (i))

(b) $3x - 2 < 4$ B1

$x < 2$

$3x - 2 > -4$ M1 ($3x - 2 > -4$)

$x > -\frac{2}{3}$ A1

$x > -\frac{2}{3}$ *and* $x < 2$ A1 (must indicate both conditions, C.A.O)

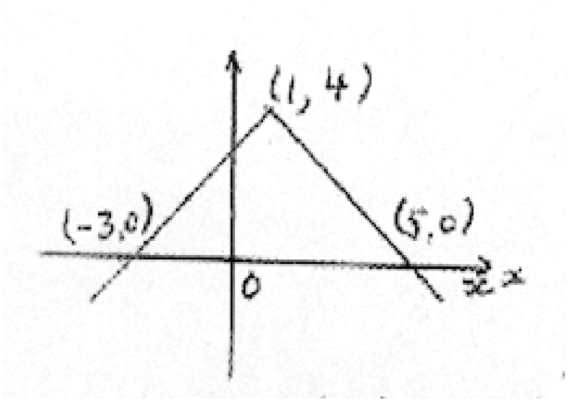
(8)

7 *(a)*

(i)	$\dfrac{(2x+3)^{\frac{3}{2}}}{\frac{3}{2}.2} = \dfrac{(2x+3)^{\frac{3}{2}}}{3}\ (+C)$	M1 $\left(\dfrac{k(2x+3)^{\frac{3}{2}}}{\frac{3}{2}}\right.$,k = 1, 2, ½)
		A1 (k = ½)
(ii)	$\frac{3}{7}\ln\lvert 7x+2\rvert$ *(+C)*	M1 $k\ln(7x+2)$
		A1 $\left(k=\frac{3}{7}\right)$
(iii)	$\frac{5}{2}e^{2x-7}\ (+C)$	M1 (ke^{2x-7})
		A1 $\left(k=\frac{5}{2}\right)$
(b)	$\left[-\frac{1}{4}\cos(4x+\frac{\pi}{6})\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$	M1($k\cos\left(4x+\frac{\pi}{6}\right)$ $k=\frac{1}{4},\ -\frac{1}{4},-1,-4$)
		A1 $\left(k=-\frac{1}{4}\right)$
	$=\frac{1}{4}\left[-\cos\left(\frac{4\pi}{3}+\frac{\pi}{6}\right)+\cos\left(\frac{4\pi}{6}+\frac{\pi}{6}\right)\right]$	
	$=\frac{1}{4}\left[-\cos\frac{3\pi}{2}+\cos\frac{5\pi}{6}\right]$	A1 $\left(k(\cos\frac{3\pi}{2}-\cos\frac{5\pi}{6}\right)$allowable k
	$=\frac{1}{4}\left(0+\left(-\frac{\sqrt{3}}{2}\right)\right)=\frac{-\sqrt{3}}{2}\approx -0.217$	A1 (C.A.O) (either form)

(10)

8.



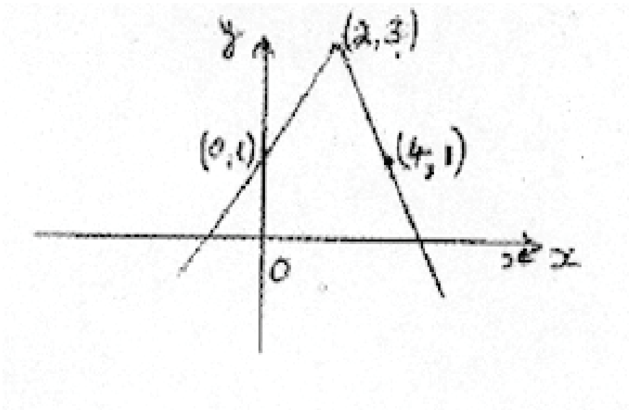
Marks conditional on inverted V shape being present,

B1 (2 correct x values),

B1 (y = 4 for highest pt.)

B1 (all correct)

S.Case for 3 correct points and incorrect or no graph, B1



B1 (left line of graph intersects y- axis at (0,1))

B1 (Second point)

B1 (all correct)

S. Case for 3 correct pts and incorrect or no graph, B1

(6)

9. *(a)*	$fg(x) = \ln e^{4x} = 4x$	M1 (correct order) A1
(b)	$gf(x) = e^{4\ln x}$	M1 (correct order)
	$= e^{\ln x^4}$	A1 (power laws)
	$= x^4$	A1
		(5)

10. *(a)*	Range is $(0, \infty)$	B1
(b)	Let y = $\frac{1}{\sqrt{x-2}}$	
	$y^2 = \frac{1}{x-2}$	M1 $\left(y^2 = \frac{1}{x-2} \right)$(and attempt to solve)
	$x - 2 = \frac{1}{y^2}$	A1
	$x = 2 + \frac{1}{y^2}$	A1 (C.A.O,)
	$f^{-1}(x) = 2 + \frac{1}{x^2}$	B1 (F.T candidate's $x = f(y)$)
Domain	$(0, \infty)$, range $(2, \infty)$	B1 (both values correct or F.T. from (a))
(c)	$2 + \frac{1}{x^2} = -\frac{3}{x}$	M1 (equating and attempting to set up quadratic equation)
	$2x^2 + 1 = -3x$	
	$2x^2 + 3x + 1 = 0$	A1 (F.T one $\pm$ slip in $f^{-1}(x)$)
	$(2x+1)(x+1) = 0$	
	$x = -\frac{1}{2}, -1$	A1
Not in domain of f^{-1} $\therefore$ No solutions		A1(F.T candidate's values)
		(10)