MATHEMATICS C3

1.	*(a)*	$h = 0{\cdot}2$	M1 (correct formula $h = 0.2$)
		Integral $\approx \frac{0\cdot 2}{3}$ [0·69314718 + 1·44456327	
		+ 4 (0·89199804 + 1·26976055)	B1 (3 values)
		+ 2(1·08518927)]	B1 (2 values)
		= 0·864	A1 (F.T. one slip)
	(b)	Second integral ≈ 0·432	B1 (F.T. answer in *(a)*)
			5
2.	*(a)*	$\theta = 0$	B1 (appropriate choice of θ)
		l.h.s. = 1	B1 (*l.h.s* ≠ *r.h.s.*)
		r.h.s. = – 1	
		$(\therefore \cos 3\theta \neq 3\cos^3\theta - 4\cos\theta)$	
	(b)	$\sec^2\theta - 1 + 2\sec\theta = 7$	M1 ($\tan^2\theta = \sec^2\theta - 1$)
		$\sec^2\theta + 2\sec\theta - 8 = 0$	M1 (correct formula for
		$(\sec\theta + 4)(\sec\theta - 2) = 0$	for $(a\sec\theta + b)(c\sec\theta + d)$ where ac = coeff of $\sec^2\theta$ bd = constant)
		$\sec\theta = -4, 2$	
		$\cos\theta = -\frac{1}{4}, \frac{1}{2}$	A1 (C.A.O.)
		θ = (104º – 105º) (255º – 256º)	B1 (104 – 105) B1 (255 – 256)
		60º, 300º	B1 (60º, 300º)
			8

3.

x	$\cos x + 2x - 2$
0	-1
$\frac{\pi}{2}$	1·14

M1(attempt to find values (or signs))

Change of sign indicates presence of root (in $(0, \frac{\pi}{2})$) — A1 (correct values (or signs) and conclusion)

$x_0 = 0·5$, $x_1 = 0·5612087$, $x_2 = 0·5766937$ — B1 (x_1)

$x_3 = 0·5808650$, $x_4 = 0·5820059$ — B1 (x_4)

Root $\approx$ 0·582

Check x = 0·5815, 0·5828

x	$\cos x + 2x - 2$	
0·5815	-0·0014	M1(attempt to find values (or signs))
0·5825	0·00009	A1 (correct)

Change of sign indicates that the root is 0·582 (correct to 3 decimal places) — A1 (F.T. one slip)

7

4. *(a)* $15(1 + 2x)^{14}.2 = 30(1 + 2x)^{14}$ — M1 ($15(1 + 2x)^{14}.k$, any k) A1 ($k = 2$, simplified result)

(b) $\frac{1}{1+x^2}.2x\left(=\frac{2x}{1+x^2}\right)$ — M1 $\left(\frac{1}{1+x^2}\times f(x), f(x)=1,2,kx\right)$ A1($f(x) = 2x$) (Final answer)

(c) $\frac{(1+\sin x)(-\sin x)-(2+\cos x)\cos x}{(1+\sin x)^2}$ — M1 $\left(\frac{(1+\sin x)f(x)-(2+\cos x)g(x)}{(1+\sin x)^2}\right)$

A1 ($f(x) = -\sin x$, $g(x) = \cos x$)

$= \frac{-1-\sin x-2\cos x}{(1+\sin x)^2}$ — A1 (simplified answer)

(d) $\frac{1}{1+(3x)^2}\times 3 \quad \left(=\frac{3}{1+9x^2}\right)$ — M1 $\left(\frac{k}{1+(3x)^2}, \text{any } k\right)$ A1 ($k = 3$ and final result)

(e) $x^2(\sec^2 x) + (2x)\tan x$ — M1 ($x^2 f(x) + g(x)\tan x$) A1 ($f(x) = \sec^2 x$, $g(x) = \tan x$)

11

5. $\left(\frac{dy}{dx} = 0\right) \quad 2e^{2x} - 1 = 0$ — M1 (attempt to find $\frac{dy}{dx}$ and set = 0)

M1 ($ke^{2x} - 1$, any k)
A1 ($k = 2$)

$e^{2x} = \frac{1}{2}$

$x = \frac{1}{2}\ln\left(\frac{1}{2}\right)$ (o.e.) — A1 (C.A.O.)

$\frac{d^2y}{dx^2} = 4e^{2x}$ — M1 (correct attempt to use any method)

$\frac{d^2y}{dx^2} = 4e^{2x} = 2 > 0$

$\therefore$ minimum point. — A1 (F.T. $\frac{d^2y}{dx^2} = ke^{2x}$)

6

Alternative:
Sign test for stationary point

$x = -0{\cdot}34,\ \frac{dy}{dx} = 0\cdot013 > 0$

$x = -0{\cdot}35,\ \frac{dy}{dx} = -0\cdot006 < 0$

$\frac{dy}{dx}$ changes from – to +

$\therefore$ minimum point

6. *(a)* $3x^2 + x^2\frac{dy}{dx} + 2xy + 4y^3\frac{dy}{dx} = 0$ — B1 ($x^2\frac{dy}{dx} + 2xy$)

B1 ($4y^3\frac{dy}{dx}$)

$\frac{dy}{dx} = -\frac{3x^2 + 2xy}{x^2 + 4y^3}$ — B1 (all correct, for final result C.A.O.)

(b) (i) $\dfrac{dy}{dx} = \dfrac{dy}{dt} \Big/ \dfrac{dx}{dt} = \dfrac{2t}{3t^2}\left(= \dfrac{2}{3t}\right)$ M1 ($\dfrac{dy}{dx} = \dfrac{\dot{y}}{\dot{x}}$)

A1 (one differentiation)
A1 (other differentiation)

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{\frac{-2}{3t^2}}{3t^2}$$ M1 (correct formula)

A1 (correct differentiation)
F.T. one slip for differentiation
of equivalent difficulty)

$= -\dfrac{2}{9t^4}$ (o.e.) A1 (C.A.O.)

9

7. *(a)* (i) $-\dfrac{1}{8(2x+3)^4}$ (+C) M1 $\left(\dfrac{k}{(2x+3)^4}, \text{any } k\right)$

A1 $\left(k = -\dfrac{1}{8}\right)$

(ii) $-\dfrac{1}{3}e^{2-3x}$ (+C) M1 (ke^{2-3x}, any k)

A1 ($k = -\dfrac{1}{3}$)

(b) $\left[2\ln(3x+2)\right]_0^2$ M1 ($k$ln (3x + 2, any k)
A1 (k = 2)

= 2 (ln 8 – ln 2) A1 (k (ln 8 – ln 2,
F.T. previous k)

= 2 ln 4

= ln 16 A1 (F.t., k allow ln $\left(\dfrac{8}{2}\right)^k$)

(c) $\left[\dfrac{1}{3}\sin\left(3x + \dfrac{\pi}{4}\right)\right]_0^{\frac{\pi}{4}}$ M1 (k sin $\left(3x + \dfrac{\pi}{4}\right)$, any +ve k)

A1 ($k = \dfrac{1}{3}$)

= $\dfrac{1}{3}\left[\sin\pi - \sin\dfrac{\pi}{4}\right]$ A1 (k(sinπ – sin $\dfrac{\pi}{4}$)

F.T. previous k

= $-\dfrac{1}{3\sqrt{2}}$ (o.e.) A1 (C.A.O.)

12

8.	First graph	B1 ($y = -4$ for stationary pt)
		B1 (shift of 3 to right, 2 correct x values)
		B1 (all correct) C.A.O.
	Second graph	M1 (maximum in second quadrant)
		A1 (correct intercept at (0, 1))
		A1 (correct st pt)
		6

9. *(a)*

Let $y = \ln(5x - 4) + 2$	B1 (attempt to isolate x, $y - 2 = ...$)	
$y - 2 = \ln(5x - 4)$		
$e^{y-2} = 5x - 4$	M1 (exponentiating)	
$x = \dfrac{e^{y-2} + 4}{5}$	A1	F.T. one slip
$f^{-1}(x) = \dfrac{e^{x-2} + 4}{5}$	A1	

(b) domain $[2, \infty)$, range $[1, \infty)$ — B1, B1

6

10. $|\,2x + 1\,| + 2 + 5 > 10$ — M1 (attempt at composition, correct order)

$x > 1$ — B1

$2x + 1 < -3$ — M1 ($2x + 1 < -3$)

$x < -2$ — A1

$x > 1$ or $x < -2$ or $(1, \alpha) \cup (-2, -2)$ — A1

For incorrect composition,

$|\,2(x + 5) + 1\,| + 2 > 10$ — M0

$x > -\frac{3}{2}$ — B0

$|2x + 11\,| < -8$ — M1

$x < -\frac{19}{2}$ — A1 (F.T.)

$x > -\frac{3}{2}$ or $x < -\frac{19}{2}$ or $\left(-\frac{3}{2}, \infty\right) \cup \left(-\infty, -\frac{19}{2}\right)$ — A1 (F.T.)

Alternatively,

$|\,2x + 1| > 3$

$(2x + 1)^2 > 9$ — M1

$x^2 + x - 2 > 0$

$(x + 2)(x - 1) > 0$

$x > 1$ — B1

$x < -2$ — M1, A1

$x > 1$ or $x < -2$ or $(1, \infty) \cup (-\infty, -2)$ — A1

5