

MATHEMATICS C3

1. (a) $h = 0.25$

$$\begin{aligned} \text{Integral} &\approx \frac{0.25}{3} [1.7320508 + 4.2426407 + \\ &4(2.1074644 + 3.3732634) \\ &+ 2(2.6575365)] \end{aligned}$$

$$\approx 2.768$$

M1 ($h= 0.25$ use of correct formula)

B1 (3 values)

B1 (2 further values)

A1 (F.T. one slip)

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2. (a) $\theta = 30^\circ$, for example
 $\tan 2\theta = \sqrt{3} \approx 1.732$

$$2 \tan \theta = \frac{2}{3} \approx 1.15$$

$$(\therefore \tan 2\theta \neq 2 \tan \theta)$$

(b) $4 (\cosec^2 \theta - 1) = 11 - 4 \cosec \theta$

$$4 \cosec^2 \theta + 4 \cosec \theta - 15 = 0$$

$$(2 \cosec \theta - 3)(2 \cosec \theta + 5) = 0$$

$$\cosec \theta = \frac{3}{2}, -\frac{5}{2}$$

$$\sin \theta = \frac{2}{3}, -\frac{2}{2}$$

$$\theta = 41.8^\circ, 138.2^\circ, 203.6^\circ, 336.4^\circ$$

M1 (substitution of θ in both,
one correct value)

A1 (both correct and
clearly unequal)

M1 (correct use of $\cosec^2 \theta = 1 + \cot^2 \theta$)

M1 (grouping terms and attempting
to solve quad. in $\cosec \theta$ or $\sin \theta$
correct formula
or $(a \cosec^2 \theta + b)(\cosec \theta + d)$
where $ac = \text{coefft of } \cosec^2 \theta$
 $bd = \text{constant term}$)

A1 (CAO)

B1 ($41.5^\circ - 42^\circ$)
B1 ($2035^\circ - 204^\circ$)
B1 ($336^\circ - 336.5^\circ$)

3. (a) $4y^3 \frac{dy}{dx} + x^3 \frac{dy}{dx} + 3x^2 y = 2x + 4$

$$4 \frac{dy}{dx} + 8 \frac{dy}{dx} + 12 = 8$$

$$\frac{dy}{dx} = -\frac{4}{12}$$

B1 ($4y^3 \frac{dy}{dx}$)

B1 ($x^3 \frac{dy}{dx} + 3x^2 y$)

B1 ($2x + 4$)

B1 (C.A.O.)

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(b) (i) $\frac{dy}{dx} = \frac{12t^3}{6t^2}$ (o.e.) M1 A1

(ii) $\frac{d^2y}{dx^2} = \frac{d}{dt} \left(\frac{dy}{dx} \right) = \frac{2}{6t^2}$ (o.e.) M1 (use of correct formula)
A1 (F.T. one slip)

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4. (a) $\frac{2}{1+x^2} - \frac{12x}{1+x^2} - 8x = 0$
B1 $\left(\frac{2}{1+x^2} \right)$
M1 $\left(\frac{kx}{1+x^2} \right)$ A1 ($k = 12$)

$2 - 12x - 8x - 8x^3 = 0$ A1 (correct unsimplified equation, F.T. one slip)

$4x^3 + 10x - 1 = 0$ A1 (convincing)

(b)
$$\begin{array}{r} x \\ 0 \\ \hline 1 & 13 \end{array}$$
 Change of sign indicates
presence of root M1 (attempt to find signs or values)
A1 (correct signs, values and conclusion)

$x_0 = 0.1, x_1 = 0.0996, x_2 = 0.0996048$ B1 (x_1)

$x_3 = 0.0996047 \approx 0.099605$ B1 (x_3 , rounded or unrounded)

Check 0.0996045, 0.0996055 M1 (attempt to find signs or values)
A1 (correct)

$$\begin{array}{r} x \\ 0.0996045 \\ \hline 0.0996055 \end{array} \quad \begin{array}{r} 4x^3 + 10x - 1 \\ -0.000002 \\ \hline 0.000008 \end{array}$$
 Root lies between 0.0996045 and 0.0996055 and is therefore 0.099605 to 6 decimal places

A1 (correct conclusion)

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5. (a) $-e^{3x} \sin x + 3e^{3x} \cos x$ M1 ($f(x)e^{3x} + g(x) \cos x$)
A1 ($f(x) = -\sin x, g(x) = ke^{3x}$)
A1 ($k = 3$, correct unambiguous answer)

(b)	$\frac{(3x^2 + 2)4x - (2x^2 + 1)6x}{(3x^2 + 2)^2}$	M1 $\left(\frac{(3x^2 + 2)f(x) - (2x^2 + 1)g(x)}{(3x^2 + 2)^2} \right)$ A1 ($f(x) = 4x$, $g(x) = 6x$)
	$= \frac{2x}{(3x^2 + 2)^2}$	A1 (C.A.O.)
(c)	$10x \sec^2(5x^2 + 3)$	M1 ($\sec^2(5x + 3)$, allow $g(x) = 1$) A1 (correct unambiguous answer)
(d)	$\frac{1}{2x} \times 2 = \frac{1}{x}$	M1 ($\frac{k}{2x}$, allow $k = 1, 2$) A1 (simplified answer)
(e)	$\frac{3}{\sqrt{1-(3x)^2}} \left(= \frac{3}{\sqrt{1-9x^2}} \right)$	M1 ($\frac{k}{\sqrt{1-(3x)^2}}$ (o.e.), allow $k = 1$) A1 ($k = 3$)

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6.	(a)	$3x - 8 \leq 5$	
		$x \leq \frac{13}{3}$	B1
		$3x - 8 \geq -5$	M1
		$x \geq 1$	
		$1 \leq x \leq \frac{3}{13}$ (or $x \geq 1$ and $x \leq \frac{3}{13}$)	A1 (must indicate both conditions apply)
	(b)	Graphs	M1 (for $ x $, V shape through origin) A1 (translation in +ve y direction, cusp at $(\pm 2, 1)$) A1 (cusp at $(-2, 1)$) A1 (correct relative positions)

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7.	(a)	(i)	$\frac{4}{7} \ln 7x + 2 - \frac{5}{6(3x + 1)^2}$	M1 ($k \ln(7x + 2)$)
			$(+C)$	A1 $\left(k = \frac{4}{7} \right)$
				M1 $\left(\frac{k}{(3x + 1)^2} \right)$ A1 $\left(k = -\frac{5}{6} \text{ (o.e.)} \right)$

(ii) $\frac{1}{2} \sin 2x (+C)$ M1 ($k \sin 2x, k = \frac{1}{2}, -\frac{1}{2}, 1, 2$)

A1 ($k = \frac{1}{2}$)

(b)
$$\left[2e^{\frac{x}{2}} \right]_0^4$$

 $= 2e^2 - 2e^0$
 ~ 12.8 M1 ($ke^{\frac{x}{2}}, k = 2, \frac{1}{2}, 1$)
A1 ($k = 2$)
M1 ($ke^2 - ke^0$, allowable ks)
A1 (C.A.O. at least 3 sig. figs)

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8. (a) Let $y = 3x^2 + 4$ M1 ($x^2 = f(y)$)

$$\begin{aligned} x^2 &= \frac{y-4}{3} \\ x &= \pm \sqrt{\frac{y-4}{3}} \end{aligned}$$

A1

Choose + \because domain of f is $x \geq 0$

$$x = \sqrt{\frac{y-4}{3}}$$

A1 (F.t. one slip)

$$f^{-1}(x) = \sqrt{\frac{x-4}{3}}$$

A1 (F.T. one slip)

Domain is $x \geq 4$, range is $f^{-1}(x) \geq 0$ (o.e.)

Domain is $x \geq 4$, range is $f^{-1}(x) > 0$

B1, B1

(b) Graphs

M1 (full or half parabola passing through (0,b))

A1 (r.h. branch and minimum at (0,4))

A1 F.T. b, full or half parabola))

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9. (a) Domain is $(2, \infty)$ B1

(b) $(fg(x) = 5)$

$$e^{\ln(x^2-4)} = 5 = 5$$

M1 (correct order)

$$x^2 - 4 = 5$$

or

$$\ln(x^2 - 4) = \ln 5$$

A1 (either)

$$x^2 = 9$$

$x = 3$ (-3 not in domain)

A1

A1 (with reason)

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