

MATHEMATICS C3

1. $h = 0.1$

M1 ($h = 0.1$ correct formula)

$$\begin{aligned} \text{Integral} &\approx \frac{0.1}{3} [0.5 + 0.4279957 + 4(0.4772563 + 0.4420154) \\ &\quad + 2(0.4582276)] \end{aligned}$$

B1 (3 values)

$$\approx 0.184$$

B1 (2 values)

A1 (F.T. one slip)

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2. (a) $\theta = 0$ for example
 $\cos \theta = 1$

B1 (choice of values and attempt to use)

$$1 - 2 \cos^2 \theta = -1$$

B1 (for correct demonstration)

$$(\therefore \cos 2\theta \neq 1 - 2 \cos^2 \theta)$$

(b) $\operatorname{cosec}^2 \theta - 1 = 7 - 2 \operatorname{cosec} \theta$

M1 (substitution of $\cot^2 \theta = \operatorname{cosec}^2 \theta - 1$)

$$\begin{aligned} \operatorname{cosec}^2 \theta + 2 \operatorname{cosec} \theta - 8 &= 0 \\ (\operatorname{cosec} \theta + 4)(\operatorname{cosec} \theta - 2) &= 0 \end{aligned}$$

M1 (attempt to solve)

$$\operatorname{cosec} \theta = -4, 2$$

$$\sin \theta = -\frac{1}{4}, \frac{1}{2}$$

A1

$$\theta = 194.5^\circ, 345.5^\circ, 30^\circ, 150^\circ$$

B1 (194.5), B1 (345.5)
B1 (30°, 150°)

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3. (a) (i) $\frac{dy}{dx} = \frac{5t^4 + 20t^2}{10t}$

M1 (attempt to use $\frac{dy}{dx} = \frac{\dot{y}}{\dot{x}}$)

A1 A1

(ii) $\frac{5t^4 + 20t^2}{10t} = 1$

M1 (use of equation and attempt to simplify)

$$\frac{t^3 + 4t}{2} = 1$$

$$t^3 + 4t - 2 = 0$$

A1 (convincing)

(b)	$\begin{array}{r} t \\ \hline 0 \\ \hline t^3 + 4t - 2 \\ \hline -2 \end{array}$	Change of sign indicates presence of root between 0 and 1	M1 (attempt to find signs or values) A1 (correct values or signs and conclusion)
	1	3	A1 (correct values or signs and conclusion)
	$t_0 = 0.5, t_1 = 0.46875$		B1 (t_1)
	$t_2 = 0.4742508, t_3 = 0.4733336, t_4 = 0.4734880$ (0.4735)		B1 (t_4 to 4 decimal places)
	Try 0.47345, 0.47355		
	$\begin{array}{r} t \\ \hline 0.47345 \\ \hline t^3 + 4t - 2 \\ \hline -0.00007 \\ 0.47355 \\ \hline 0.0004 \end{array}$		M1 (attempt to find signs or values) M1 (correct values or signs)
	Change of sign indicates root is 0.4735 (correct to 4 decimal places)		A1

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4.	(a)	Graph	B1
		Graph	M1 (shape) A1 ((0, 4)) A1 ((± 2, 0))
(b)		$5x - 3 > 4$ $x > \frac{7}{5}$	B1
	or	$5x - 3 < -4$ $x < -\frac{1}{5}$	M1 A1 (must have 'or' in either part) (o.e.)
	<u>Alternatively</u> $(5x - 3)^2 > 16$		M1 (forming quadratic and attempting to solve)
	$25x^2 - 30x - 7 > 0$ $(5x + 1)(5x - 7) > 0$ $-\frac{1}{5}, \frac{7}{5}$		A1 (fixed points)
	$x < -\frac{1}{5}$ or $x > \frac{7}{5}$	(o.e.)	A1 (F.T. fixed points)

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5. $6y \frac{dy}{dx} + 2xy^3 + 3x^2y^2 \frac{dy}{dx} + 4x^3 - 2x = 0$

$\frac{dy}{dx} = -4$

B1 ($6y \frac{dy}{dx}$)
B1 ($2xy^3 + 3x^2y^2 \frac{dy}{dx}$)
B1 (correct differentiation of x^4, x^2)
B1 (F.T. one slip)

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6. (a) (i) $x^2 \cos x + 2x \sin x$

(ii) $\frac{2x}{x^2 + 3}$

(iii) $-2e^{9-2x}$

(iv) $-\frac{24}{(3x+7)^3}$

(v) $-\frac{24}{\sqrt{1-(3x)^2}}$ (o.e.)

M1 ($x^2 f(x) + \sin x g(x)$)
A1 ($f(x) = \cos x, g(x) = 2x$)

M1 ($\frac{f(x)}{x^2 + 3}, f(x) \neq 1$), A1

M1 (ke^{9-2x}) A1 ($k = -2$)

M1 ($\frac{k}{(3x+7)^3}$, allow unsimplified)
A1 (simplified answer)

M1 ($\frac{k}{\sqrt{1-(3x)^2}}$) A1 ($k = 3$)

(S. Case allow B1 for $\frac{3}{\sqrt{1-3x^2}}$)

(b) $\frac{dy}{dx} = \frac{(1-\tan x)(\sec^2 x) - (1+\tan x)(-\sec^2 x)}{(1-\tan x)^2}$

$= \frac{2\sec^2 x}{(1-\tan x)^2}$

which is positive since $\sec^2 > 0, (1-\tan x)^2 > 0$

M1($((1-\tan x)f(x)) - \frac{(1+\tan x)g(x)}{(1+\tan x)^2}$)
A1 ($f(x) = \sec^2 x, g(x) = -\sec^2 x$)
A1 (simplified)

B1 (convincing)

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7. (a) (i) $-\frac{1}{2} \ln |5-2x| (+ C)$

M1 A1

(ii) $\frac{(3x+2)^{21}}{63} (+ C)$ M1 ($k(3x+2)^{21}$)
A2($k=\frac{1}{63}$) or A1 ($k=\frac{1}{21}$ or $\frac{1}{3}$)

(iii) $\frac{1}{7}e^{7x} (+ C)$ M1 (ke^{7x}) A1 ($k=\frac{1}{7}$)

(b)
$$\int_0^{\frac{\pi}{3}} \cos(3x + \frac{\pi}{3}) dx$$

 $= \left[\frac{1}{3} \sin(3x + \frac{\pi}{3}) \right]_0^{\frac{\pi}{3}}$ M1 [$k \sin(3x + \frac{\pi}{3})$, allow
 $k = 1, \frac{1}{3}, -\frac{1}{3}, 3$]
A1 ($k = \frac{1}{3}$)
 $= \frac{1}{3} \sin\left(\frac{4\pi}{3}\right) - \frac{1}{3} \sin\frac{\pi}{3}$ m1 (F.T. $k(\sin \frac{4\pi}{3} - \sin \frac{\pi}{3})$)
 $= -\frac{2}{3} \sin \frac{\pi}{3}$
 $= -\frac{\sqrt{3}}{3}$ or -0.577 correct to 3 decimal places $-(0.578)$ A1 (C.A.O.)

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8.	(a) Range of $f[1, \infty)$, Range of $g [1, \infty)$	B1, B1
(b)	$gf(x) = (e^x)^2 + 1$	M1 (correct order of composition)
	$= e^{2x} + 1$	A1
(c)	domain of gf	[0, ∞) B1
	range of gf	[2, ∞) B1
(d)	Graph	$y = e^x$ $y = e^{2x} + 1$ B1 (full curve with (0, 1)) B1 (truncated) B1 (0, 2) B1 (steeper curve) B1 (truncated)

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9. (a) Let $y = \frac{8}{x+2}$

$$(x+2)y = 8$$

M1 (attempt to isolate x)

$$x+2 = \frac{8}{y}$$

$$x = \frac{8}{y} - 2$$

A1

$$f^{-1}(x) = \frac{8}{x} - 2$$

A1 (F.T.)

(b) domain $(0, 4]$

B1

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MATHEMATICS C4

1. (a) Let $\frac{x+3}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1}$
- $\therefore x+3 \equiv Ax(x-1) + B(x-1) + Cx^2$
- $x=1$ $4 = C, \quad C = 4$ M1 (correct form)
- $x=0$ $3 = -B, \quad B = -3$ M1 (clearing fractions and attempt to solve)
- Equate coefficients of x^2 $0 = A + C, \quad A = -4$ A1 (2 constants)
- No need for display A1 (other constant)
(F.T. if 2 Ms scored)

(b)
$$\int -\frac{4}{x} dx - \int \frac{3}{x^2} dx + \int \frac{4}{x-1} dx$$

$$= -4 \ln(x) + \frac{3}{x} + 4 \ln|x-1| \quad (\text{o.e.}) \quad \text{B1, B1 (two logs)}$$

$$(+ C)$$

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2. $5x^4 + y^2 + 2xy \frac{dy}{dx} + 3y^2 \frac{dy}{dx} = 0$
- $\frac{dy}{dx} = -\frac{2}{3}$ (o.e.) B1 ($y^2 + 2xy \frac{dy}{dx}$)
 $\frac{dy}{dx}$ B1 ($3y^2 \frac{dy}{dx}$)
- Equation is $y - 3 = -\frac{2}{3}(x + 1)$ B1 (F.T. candidate's $\frac{dy}{dx}$)

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3. $4 \cos x + 2 \sin x = R \cos(x - \alpha)$
 $R \cos \alpha = 4, R \sin \alpha = 2$
 $R = \sqrt{20}, \alpha = \tan^{-1}\left(\frac{1}{2}\right) = 26.6^\circ$
- $\cos(x - 26.6^\circ) = \frac{3}{\sqrt{20}}$ M1 ($\cos(x \pm \alpha)$)
 $x - 26.6^\circ = 47.9^\circ, 312.1^\circ$ B1 ($\sqrt{20}$)
- $x = 74.4^\circ, 338.7^\circ$ (C.A.O.) A1 (correct α for given presentation)
 $\cos(x \pm 26.6^\circ) = \frac{3}{\sqrt{20}}))$
 $A1 (\text{for one value}) \quad (\text{C.A.O.})$
- (accept $74, 75, 338^\circ, 339^\circ$)
 $A1, A1$

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