C3

1.

0	0·5
0·2	0·401312339
0·4	0·310025518
0·6	0·231475216 (3 values correct) B1
0·8	0·167981614 (5 values correct) B1

Correct formula with $h = 0{\cdot}2$ M1

$I \approx \frac{0{\cdot}2}{3} \times \{0{\cdot}5 + 0{\cdot}167981614 + 4(0{\cdot}401312339 + 0{\cdot}231475216) + 2(0{\cdot}310025518)\}$

$I \approx 0{\cdot}2 \times 3{\cdot}819182871 \div 3$

$I \approx 0{\cdot}254612191$

$I \approx 0{\cdot}2546$ (f.t. one slip) A1

Note: Answer only with no working earns 0 marks

2. (*a*) e.g. $\theta = \frac{\pi}{2}$

$\cos\theta + \cos 4\theta = 1$ (choice of θ and one correct evaluation) B1

$\cos 2\theta + \cos 3\theta = -1$ (both evaluations correct but different) B1

(*b*) $2(\sec^2\theta - 1) = \sec\theta + 8$ (correct use of $\tan^2\theta = \sec^2\theta - 1$) M1

An attempt to collect terms, form and solve quadratic equation in $\sec\theta$, either by using the quadratic formula or by getting the expression into the form $(a\sec\theta + b)(c\sec\theta + d)$, with $a \times c$ = coefficient of $\sec^2\theta$ and $b \times d$ = constant m1

$2\sec^2\theta - \sec\theta - 10 = 0 \Rightarrow (2\sec\theta - 5)(\sec\theta + 2) = 0$

$\Rightarrow \sec\theta = \frac{5}{2}, \sec\theta = -2$

$\Rightarrow \cos\theta = \frac{2}{5}, \cos\theta = -\frac{1}{2}$ (c.a.o.) A1

$\theta = 66{\cdot}42°, 293{\cdot}58°$ B1

$\theta = 120{\cdot}0°, 240{\cdot}0°$ B1 B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.
$\cos\theta = +, -$, f.t. for 3 marks, $\cos\theta = -, -$, f.t. for 2 marks
$\cos\theta = +, +$, f.t. for 1 mark

3. (*a*) $\frac{d}{dx}(y^4) = 4y^3\frac{dy}{dx}$ B1

$\frac{d}{dx}(4x^2y) = 4x^2\frac{dy}{dx} + 8xy$ B1

$\frac{d}{dx}(3x^3 - 5x) = 9x^2 - 5$ B1

$\frac{dy}{dx} = \frac{9x^2 - 5 - 8xy}{4y^3 + 4x^2}$ (c.a.o.) B1 (*b*) $\frac{dx}{dt} = 4 - 2\sin 2t$, B1

$\frac{dy}{dt} = 3\cos 3t$ B1

Use of $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ M1

Substituting $\frac{\pi}{12}$ for t in expression for $\frac{dy}{dx}$ m1

$\frac{dy}{dx} = \frac{1}{\sqrt{2}}$ A1

4. $f(x) = 4x^3 - 2x - 5$

An attempt to check values or signs of $f(x)$ at $x = 1$, $x = 2$ M1

$f(1) = -3 < 0$, $f(2) = 23 > 0$

Change of sign $\Rightarrow f(x) = 0$ has root in (1, 2) A1

$x_0 = 1{\cdot}2$

$x_1 = 1{\cdot}227601026$ (x_1 correct, at least 5 places after the point) B1

$x_2 = 1{\cdot}230645994$

$x_3 = 1{\cdot}230980996$

$x_4 = 1{\cdot}231017841 = 1{\cdot}23102$ (x_4 correct to 5 decimal places) B1

An attempt to check values or signs of $f(x)$ at $x = 1{\cdot}231015$, $x = 1{\cdot}231025$ M1

$f(1{\cdot}231015) = -1{\cdot}197 \times 10^{-4} < 0$, $f(1{\cdot}231025) = 4{\cdot}218 \times 10^{-5} > 0$ A1

Change of sign $\Rightarrow \alpha = 1{\cdot}23102$ correct to five decimal places A1

Note: 'Change of sign' must appear at least once.

5. (*a*) (i) $\frac{dy}{dx} = 13 \times (7 + 2x)^{12} \times f(x)$, $(f(x) \neq 1)$ M1

$\frac{dy}{dx} = 26 \times (7 + 2x)^{12}$ A1

(ii) $\frac{dy}{dx} = \frac{5}{\sqrt{1-(5x)^2}}$ **or** $\frac{1}{\sqrt{1-(5x)^2}}$ **or** $\frac{5}{\sqrt{1-5x^2}}$ M1

$\frac{dy}{dx} = \frac{5}{\sqrt{1-25x^2}}$ A1

(iii) $\frac{dy}{dx} = x^3 \times f(x) + e^{4x} \times g(x)$ M1

$\frac{dy}{dx} = x^3 \times f(x) + e^{4x} \times g(x)$ (either $f(x) = 4e^{4x}$ or $g(x) = 3x^2$) A1

$\frac{dy}{dx} = x^3 \times 4e^{4x} + e^{4x} \times 3x^2$ (all correct) A1 (*b*) $\frac{d}{dx}(\tan x) = \frac{\cos x \times m \cos x - \sin x \times k \sin x}{\cos^2 x}$ $(m = \pm 1, k = \pm 1)$ M1

$\frac{d}{dx}(\tan x) = \frac{\cos x \times \cos x - \sin x \times (-\sin x)}{\cos^2 x}$ A1

$\frac{d}{dx}(\tan x) = \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$

$\frac{d}{dx}(\tan x) = \frac{1}{\cos^2 x} = \sec^2 x$ (convincing) A1

6. (*a*) (i) $\int (7x - 9)^{1/2}\, dx = k \times \frac{(7x - 9)^{3/2}}{3/2} + c$ $(k = 1, 7, {}^1/_7)$ M1

$\int (7x - 9)^{1/2}\, dx = {}^1/_7 \times \frac{(7x - 9)^{3/2}}{3/2} + c$ A1

(ii) $\int e^{x/6}\, dx = k \times e^{x/6} + c$ $(k = 1, 6, {}^1/_6)$ M1

$\int e^{x/6} dx = 6 \times e^{x/6} + c$ A1

(iii) $\int \frac{4}{5x - 1}\, dx = 4 \times k \times \ln|5x - 1| + c$ $(k = 1, 5, {}^1/_5)$ M1

$\int \frac{4}{5x - 1}\, dx = 4 \times {}^1/_5 \times \ln|5x - 1| + c$ A1

(*b*) $\int (3x - 4)^{-3}\, dx = k \times \frac{(3x - 4)^{-2}}{-2}$ $(k = 1, 3, {}^1/_3)$ M1

$\int_2^4 8 \times (3x - 4)^{-3}\, dx = \left[8 \times {}^1/_3 \times \frac{(3x - 4)^{-2}}{-2}\right]_2^4$ A1

Correct method for substitution of limits M1

$\int_2^4 8 \times (3x - 4)^{-3}\, dx = \frac{5}{16} = 0{\cdot}3125$ (f.t. for $k = 1, 3$ only) A1

7. (*a*) Trying to solve either $3x + 1 \leq 5$ or $3x + 1 \geq -5$ M1

$3x + 1 \leq 5 \Rightarrow x \leq {}^4/_3$

$3x + 1 \geq -5 \Rightarrow x \geq -2$ (both inequalities) A1

Required range: $-2 \leq x \leq {}^4/_3$ (f.t. one slip) A1

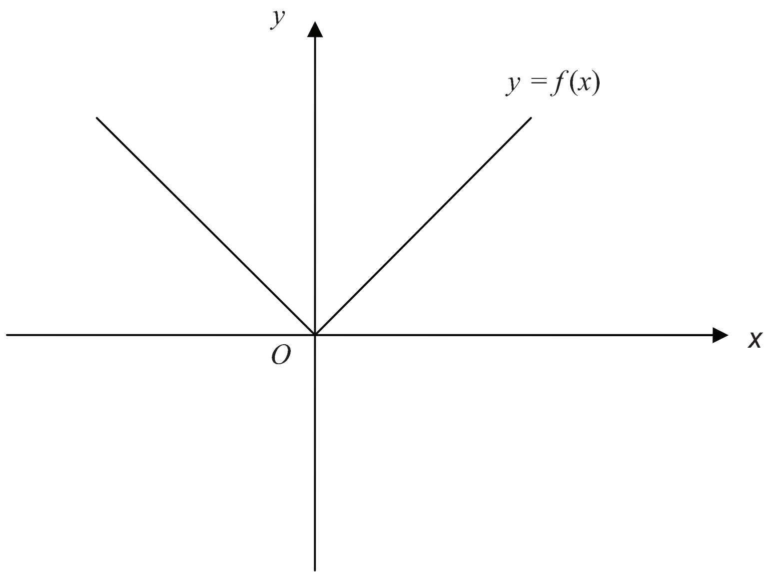
Alternative mark scheme

$(3x + 1)^2 \leq 25$ (forming and trying to solve quadratic) M1

Critical points $x = -2$ and $x = {}^4/_3$ A1

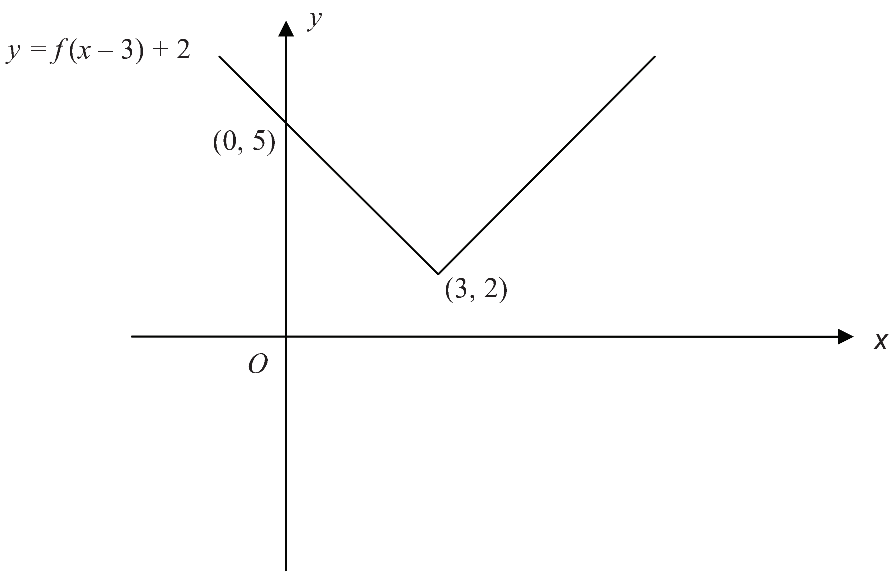
Required range: $-2 \leq x \leq {}^4/_3$ (f.t. one slip in critical points) A1

(*b*) (i)



Correct graph B1

(ii)



Translation of graph of $f(x) = |x|$ with vertex at $(\pm 3, \pm 2)$ M1

Coordinates of vertex = (3, 2) A1

Crosses y-axis at (0, 5) A1

8. (*a*) $g'(x) = \frac{3 \times f(x)}{4x^2 + 9} + 2 \quad f(x) \neq 1$ M1

$g'(x) = \frac{3 \times 8x}{4x^2 + 9} + 2$ A1

$g'(x) = \frac{24x + 8x^2 + 18}{4x^2 + 9} = \frac{2(2x + 3)^2}{4x^2 + 9}$ (convincing) A1

(*b*) (i) At stationary point, $\frac{2(2x + 3)^2}{4x^2 + 9} = 0$

or $\frac{3 \times 8x}{4x^2 + 9} + 2 = 0$ M1

$\frac{2(2x + 3)^2}{4x^2 + 9} = 0$ only when $x = -{}^3/_2$ A1

(ii) $g'(x) > 0$ either side of $x = -{}^3/_2$ (or at all other points) M1

Stationary point is a point of inflection A1

9. (*a*) $y - 5 = \ln (3x - 2)$ B1

An attempt to express candidate's equation as an exponential equation M1

$x = \frac{(e^{y-5} + 2)}{3}$ (f.t. one slip) A1

$f^{-1}(x) = \frac{(e^{x-5} + 2)}{3}$ (f.t. one slip) A1

(*b*) $D(f^{-1}) = [5, \infty)$ B1

10. (*a*) $R(f) = [1, \infty)$ B1

$R(g) = [-3, \infty)$ B1

(*b*) $gf(x) = 2\sqrt{(x + 4)^2} - 3$. M1

$gf(x) = 2x + 5$ A1

(*c*) $fg(x) = \sqrt{(2x^2 - 3 + 4)}$ (correct composition) B1

$[fg(x)]^2 = 17^2$ (candidate's $fg(x)$) M1

$x^2 = 144$ (f.t. one numerical slip) A1

$x = \pm 12$ (c.a.o.) A1

C4

1. (*a*) $f(x) \equiv \frac{A}{x} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2}$ (correct form) M1

$8 - x - x^2 \equiv A(x-2)^2 + Bx(x-2) + Cx$

(correct clearing of fractions and genuine attempt to find coefficients) m1

$A = 2$, $C = 1$, $B = -3$ (2 coefficients, c.a.o.) A1

(third coefficient, f.t. one slip in enumeration of other 2 coefficients) A1

(*b*) $f'(x) = \frac{-2}{x^2} + \frac{3}{(x-2)^2} - \frac{2}{(x-2)^3}$ (at least one of first two terms) B1

(third term) B1

(f.t. candidates values for A, B, C)

$f'(1) = 3$ (c.a.o.) B1

2. $10x + 4x\frac{dy}{dx} + 4y - 3y^2\frac{dy}{dx} = 0$

$\left[4x\frac{dy}{dx} + 4y\right]$ B1

$\left[-3y^2\frac{dy}{dx}\right]$ B1

$\frac{dy}{dx} = \frac{1}{4}$ (o.e.) (c.a.o.) B1

Use of $\text{grad}_{normal} \times \text{grad}_{tangent} = -1$ M1

Equation of normal: $y - (-2) = -4(x - 1)$

$\left[\text{f.t. candidate's value for } \frac{dy}{dx}\right]$ A1

3. (*a*) $2(2\cos^2\theta - 1) = 9\cos\theta + 7$

(correct use of $\cos 2\theta = 2\cos^2\theta - 1$) M1

An attempt to collect terms, form and solve quadratic equation in $\cos\theta$, either by using the quadratic formula or by getting the expression into the form $(a\cos\theta + b)(c\cos\theta + d)$, with $a \times c$ = coefficient of $\cos^2\theta$ and $b \times d$ = constant m1

$4\cos^2\theta - 9\cos\theta - 9 = 0 \Rightarrow (4\cos\theta + 3)(\cos\theta - 3) = 0$

$\Rightarrow \cos\theta = \frac{-3}{4}$, ($\cos\theta = 3$) (c.a.o.) A1

$\theta = 138{\cdot}59°, 221{\cdot}41°$ B1 B1

Note: Subtract (from final two marks) 1 mark for each additional root in range from $4\cos\theta + 3 = 0$, ignore roots outside range.
$\cos\theta = -$, f.t. for 2 marks, $\cos\theta = +$, f.t. for 1 mark