

MATHEMATICS C4

1. (a) Let $\frac{1}{x^2(2x-1)} \equiv \frac{A}{x} + \frac{B}{x^2} + \frac{C}{2x-1}$	M1 (Correct form)
$1 \equiv Ax(2x-1) + B(2x-1) + Cx^2$	M1 (correct clearing and attempt to substitute)
$\begin{array}{lcl} x=0 & 1=B(-1) & \therefore B=-1 \\ x=\frac{1}{2} & 1=C\frac{1}{4} & \therefore C=4 \\ x^2 & 0=2A+C & \therefore A=-2 \end{array}$	A1 (2 constants C.A.O.) A1 (third constant, F.T. one slip)
(no need for display)	
(b) $\int \left(\frac{-2}{x} - \frac{1}{x^2} + \frac{4}{2x-1} \right) dx$ $= -2 \ln x + \frac{1}{x} + 2 \ln 2x-1 $ $(+C)$	B1,B1.B1
$2x + x \frac{dy}{dx} + y + 4y \frac{dy}{dx} = 0$	B1 ($x \frac{dy}{dx} + y$) B1 ($4y \frac{dy}{dx}$)
$\frac{dy}{dx} = 5$	B1 (C.A.O.)
Gradient of normal $= -\frac{1}{5}$	M1 ($\frac{-1}{candidate's \frac{dy}{dx}}$, numerical value)
Equation of normal is $y-1 = -\frac{1}{5}(x+3)$	A1 (F.T. candidate's value)
3. (a) $R \sin \alpha = 2, R \cos \alpha = 3$ $R = \sqrt{13}, \alpha = \tan^{-1}\left(\frac{2}{3}\right) = 33.7 \text{ or } 34$	B1 ($R = \sqrt{13}$) M1 (correct method for α) A1 ($\alpha = 34^\circ$)
(b) $\cos(x-33.7^\circ) = \frac{1}{\sqrt{13}}$ $x-33.7^\circ = 73.9^\circ, 286.1^\circ$ $x = 107.6^\circ, 319.8^\circ$	B1 (one value) B1, B1

4. Volume = $\pi \int_1^4 \left(x + \frac{3}{\sqrt{x}} \right)^2 dx$ B1

$$= \pi \int_1^4 \left(x^2 + 6\sqrt{x} + \frac{9}{x} \right) dx$$

M1 (attempt to square, at least 2 correct terms)
A1 (all correct)

$$= \pi \left[\frac{x^3}{3} + 4x^{\frac{3}{2}} + 9 \ln x \right]_1^4$$

A3 (integration of 3 terms,
F.T. similar work
 $Ax^2 + B\sqrt{x} + \frac{C}{x}$)

$$= \pi [49 + 9 \ln 4] \approx 193.1$$

or 61.48π A1 (C.A.O.)

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5. (a) $\frac{dy}{dx} = \frac{-2 \sin 2t}{4 \cos t}$ M1 ($\frac{dy}{dx} = \frac{y}{x}$)
B1 ($4 \cos t$)
M1 ($k \sin 2t, k = -1, \pm 2, -\frac{1}{2}$)
A1 ($k = -2$)
M1 (correct use of formula)
A1 (C.A.O.)

(b) Equation of tangent is $y - \cos 2p = -\sin p(x - 4 \sin p)$ M1 ($y - y_1 = m(x - x_1)$)

$$x \sin p + y = \cos 2p + 4 \sin^2 p$$

$$= 1 - 2 \sin^2 p + 4 \sin^2 p$$

M1 (attempt to use correct formula)
A1

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6. (a) $\int (3x+1)e^{2x} dx = (3x+1)\frac{e^{2x}}{2} - \int \frac{3}{2}e^{2x} dx$

M1 ($f(x)(3x+1) - \int 3f(x)dx$)

$= (3x+1)\frac{e^{2x}}{2} - \frac{3e^{2x}}{4} (+ C)$

A1 ($f(x) = ke^{2x}, k = 1, \frac{1}{2}, 2$)

A1 ($k = \frac{1}{2}$)

A1 (F.T. one slip)

(b) $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sqrt{9 - 9 \sin^2 \theta} \cdot 3 \cos \theta d\theta$

B1 (for first line, unsimplified)

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 9 \cos^2 \theta d\theta$

B1 (simplified without limits)

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \left(\frac{1 + \cos 2\theta}{2} \right) d\theta$

B1 (limits)

M1 ($\cos^2 \theta = a + b \cos 2\theta$)

$= \left[\frac{9\theta}{2} + \frac{9 \sin 2\theta}{4} \right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$

A1 ($a = b = \frac{1}{2}$)

M1 ($k \sin 2\theta, k = \pm \frac{b}{2}, 2b, b$)

$= \frac{3\pi}{2} - \frac{9\sqrt{3}}{8} \quad \text{or} \quad 2.764$

A1 (C.A.O.)

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7.	(a)	$\frac{dW}{dt} = kW \quad (k > 0)$	B1
	(b)	$\int \frac{dW}{W} = \int k dt$	M1 (attempt to separate variables)
		$\ln W = kt + C$	A1 (allow absence of C)
		$t = 0, \quad W = 0.1, \quad C = \ln 0.1$	B1 (value of C)
		$\ln \frac{W}{0.1} = kt$	M1 (use of logs or exponentials)
		$\frac{W}{0.1} = e^{kt}$	
		$k = 3.0007$	B1 (value of k)
		$W = 0.1e^{3t}$	A1
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8.	(a)(i)	$\mathbf{AB} = \mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$	B1 (\mathbf{AB})
	(ii)	Equation of AB is $\mathbf{r} = 4\mathbf{i} - \mathbf{j} + \mathbf{k} + \lambda(\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$	M1 (reasonable attempt to write equations) A1 (must contain \mathbf{r} , F.T. candidate's \mathbf{AB})
	(b)	(Point of intersection is on both lines) Equate coeffs of \mathbf{i} and \mathbf{j} (any two of $\mathbf{i}, \mathbf{j}, \mathbf{k}$)	M1 (attempt to write equations using candidate's equation) A1 (2 correct equations, F.T. candidate's equations)
		$1+ = 4+\lambda$	M1 (attempt to solve)
		$\lambda = \frac{1}{3} \left(= \frac{10}{3} \right)$	A1 (C.A.O.)
		Position vector is $\frac{13}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{1}{3}\mathbf{k}$	A1 (F.T. value of λ or)
	(c)	angle between $\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ and $\mathbf{i} - \mathbf{j} + \mathbf{k}$ is required	B1 (coeffs of λ and)
		$ \mathbf{i} + 2\mathbf{j} - 2\mathbf{k} = 3, \quad \mathbf{i} - \mathbf{j} + \mathbf{k} = \sqrt{3}$	B1 (for one modulus)
		$(\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}) \cdot (\mathbf{i} - \mathbf{j} + \mathbf{k}) = \mathbf{i} \times \mathbf{i} - \mathbf{j} + \mathbf{k} \cos \theta$	M1 (use of correct formula)
		$1 - 2 - 2 = 3\sqrt{3} \cos \theta$	B1 (l.h.s. unsimplified)
		$\theta = 125.3$	A1 (C.A.O.)

9.

$$\begin{aligned}
 (1+3x)(1-2x)^{-\frac{1}{2}} &= (1+3x)\left(1 + \left(-\frac{1}{2}\right)(-2x) + \left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\frac{1}{2!}(-2x)^2 + \dots\right) \\
 &= (1+3x)\left(1 + x + \frac{3}{2}x^2 + \dots\right) \\
 &= 1 + 4x + \frac{9x^2}{2} + \dots
 \end{aligned}$$

B1, B1 (unimplified)
B1 ($1+4x$)
B1 ($\frac{9x^2}{2}$)

Exparsim is valid for $|x| < \frac{1}{2}$

B1

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10.

$$\begin{aligned}
 (x^2 + 49 < 14x) \\
 x^2 - 14x + 49 < 0 \\
 (x-7)^2 < 0 \\
 x-7 \text{ is not real} \\
 \text{contradiction} \\
 \left(\therefore x + \frac{49}{x} \geq 14 \quad \text{for all real and positive } x\right)
 \end{aligned}$$

B1
B1
B1
B1 (accept impossibility)

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