MATHEMATICS C4

1. *(a)* Let $\frac{x+3}{x^2(x-1)} \equiv \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1}$ — M1 (correct form)

$\therefore\ x + 3 \equiv Ax(x-1) + B(x-1) + Cx^2$ — M1 (clearing fractions and attempt to solve)

$\underline{x=1}$	$4 = C,$	$C = 4$	A1 (2 constants)
$\underline{x=0}$	$3 = -B,$	$B = -3$	A1 (other constant
Equate coefficients of x^2	$0 = A + C,$	$A = -4$	(F.T. if 2 Ms scored)

No need for display

(b) $\int -\frac{4}{x}dx - \int \frac{3}{x^2}dx + \int \frac{4}{x-1}dx$

$= -4\,ln(x) + \frac{3}{x} + 4\,ln\,|x-1|$ (o.e.) — B1, B1 (two logs)

$(+\,C)$

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2. $5x^4 + y^2 + 2xy\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0$ — B1 ($y^2 + 2xy\frac{dy}{dx}$)

B1 ($3y^2\frac{dy}{dx}$)

$\frac{dy}{dx} = -\frac{2}{3}$ (o.e.) — B1 (C.A.O.)

Equation is $y - 3 = -\frac{2}{3}(x+1)$ — B1 (F.T. candidate's $\frac{dy}{dx}$)

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3. $4\cos x + 2\sin x = R\cos(x - \alpha)$ — M1 (for $R\cos(x \pm \alpha)$)

$R\cos\alpha = 4$, $R\sin\alpha = 2$ — B1 $\left(\sqrt{20}\right)$

$R = \sqrt{20}$, $\alpha = \tan^{-1}\left(\frac{1}{2}\right) = 26.6°$ — A1 (correct α for given presentation)

$\cos(x - 26.6°) = \frac{3}{\sqrt{20}}$ — M1 ($\cos(x \pm 26.6°) = \frac{3}{\sqrt{20}}$)

$x - 26.6° = 47.9°, 312.1°$ — A1 (for one value) (C.A.O.)

$x = 74.4°, 338.7°$ (C.A.O.) — (accept 74, 75, 338°, 339°) A1, A1

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4. $(1+4x)^{\frac{1}{2}} - \dfrac{1}{1+3x}$

$= 1 + \left(\dfrac{1}{2}\right)(4x) + \left(\dfrac{1}{2}\right)\left(-\dfrac{1}{2}\right)\dfrac{1}{2!}(4x)^2 + \ldots$ — B2 (−1) each error

$- (1 - 3x + \dfrac{(-1)(-2)}{2}(3x)^2 + \ldots)$ — B2 (−1) each error

(correct expansion of $(1 + 3x)^{-n}$)

$= 1 + 2x - 2x^2 + \ldots$

$- 1 + 3x - 9x^2 + \ldots$ — B2

$= 5x - 11\,x^2 + \ldots$ — (−1 each error)

Expansion valid for $|x| < \dfrac{1}{4}$ (o.e.) — B1

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5. Volume $= \pi \displaystyle\int_0^1 (e^{2x} + 1)\ \ dx$ — B1 (with or without limits, after squaring $\sqrt{\ }$)

$= \pi \left[\dfrac{e^{2x}}{2} + x\right]_0^1$ — B1 (correct integration)

$= \pi \left[\dfrac{e^{2x}}{2} + 1 - \dfrac{1}{2}\right]$ — M1 (correct use of limits after attempted integration)

$\approx$ 13·177 — A1 (C.A.O.)

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6. *(a)* $\dfrac{dy}{dx} = \dfrac{2t}{2} = t$ — M1 (correct attempt to find gradient)

Gradient of normal = $-\dfrac{1}{t}$ — A1

Equation is

$y - p^2 = -\dfrac{1}{p}(x - 2p)$ — M1 ($y - y_1 = m(x - x_1)$) o.e.

$py - p^3 = -x + 2p$

$x + py = p^3 + 2p$ — A1 (convincing)

(b) A $y = 0$, $x = p^3 + 2p$ — B1 (must be correct for A and B)

B $x = 0$, $y = p^2 + 2$ — B1

$p^3 + 2p = 2(p^2 + 2)$

$p(p^2 + 2) = 2(p^2 + 2)$ — M1 (candidate's OA = k × candidate's OB, $k = \dfrac{1}{2}$ or 2)

$p = 2$ — A1 (C.A.O.)

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7. *(a)* $\displaystyle\int x^2 \ln x\,dx = \frac{x^3 \ln x}{3} - \int \frac{x^3}{3}.\frac{1}{x}dx$ — M1 (Parts and correct choice of u, v)

$\displaystyle = \frac{x^3 \ln x}{3} - \int \frac{x^2}{3}dx$ — A1

M1 (division)

$\displaystyle = \frac{x^3 \ln x}{3} - \frac{x^3}{9}$ (+ C) — A1 (C.A.O.)

(b) $dx = 2\cos\theta d\theta$

When $x = 0$, $\theta = 0$

$x = \sqrt{2}$, $\sin\theta = \dfrac{1}{\sqrt{2}}$, $\theta = \dfrac{\pi}{4}$ — B1

$\displaystyle\int_0^{\frac{\pi}{4}} \frac{4\sin^2\theta}{\sqrt{4 - 4\sin^2\theta}}.2\cos\theta d\theta$ — M1 (substitution for dx and x)

A1 (any limits)

$\displaystyle = \int_0^{\frac{\pi}{4}} \frac{4\sin^2\theta}{2\sqrt{1 - \sin^2\theta}}.2\cos\theta d\theta$

$= \int_0^{\frac{\pi}{4}} \frac{4\sin^2\theta 2\cos\theta d\theta}{2\cos\theta}$

$= \int_0^{\frac{\pi}{4}} 4\sin^2\theta d\theta$ A1 (convincing, proof of $k = 4$, any limits)

$\int = \int_0^{\frac{\pi}{4}} 2(1-\cos 2\theta)d\theta$ M1 ($a + b\cos 2\theta$)

A1 ($a = \frac{k}{2}$, $b = -\frac{k}{2}$)

$= [2\theta - \sin 2\theta]_0^{\frac{\pi}{4}}$ A1 (correct integration of two terms)

$= \frac{\pi}{2} - 1 (\approx 0\cdot 571)$ A1 (either answer, C.A.O.)

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8. *(a)* $\frac{dP}{dt} = kP$ B1

(b) $\int \frac{dP}{P} = \int kdt$ M1 (Separation of variables and ∫)

$ln\ P = kt + C$ A1

$t = 0, P = 50$ F.T. right hand side)

$ln\ 50 = C$ M1 (attempt to find C)

$ln\ P - ln\ 50 = kt$

$ln\ \frac{P}{50} = kt$ M1 (combination of logs and attempt to exponentiate)

$\frac{P}{50} = e^{kt}$

$P = 50e^{kt}$ A1

(c) $65 = 50\ e^{7k}$ M1 (taking logs correctly)

$\ln \frac{65}{50} = 7k$ $k = 0.03748$ A1

After sixteen years, $P = 50 \exp(\cdot 03748 \times 16)$ M1 (use of formula)

$\approx$ £91 (nearest pound) A1 (C.A.O.)

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9.	*(a)*	(i)	$\mathbf{AB} = 3\mathbf{i} + 6\mathbf{j} + \mathbf{k} - (\mathbf{i} + 3\mathbf{j} - 2\mathbf{k})$		M1 ($\mathbf{b} - \mathbf{a}$)
			$= 2\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$		A1
		(ii)	Equation of AB is		M1 ($\mathbf{a} + \lambda(\mathbf{b} - \mathbf{a})$)
			$\mathbf{r} = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k} + \lambda(2\mathbf{i} + 3\mathbf{j} + 3\mathbf{k})$	(o.e.)	A1 (must involve **r** or **OP** = ... F.T. **AB**)
		(iii)	(The point lies on both lines)		
			$\mathbf{i} + 3\mathbf{j} - 2\mathbf{k} + \lambda(2\mathbf{i} + 3\mathbf{j} + 3\mathbf{k})$		
			$= 2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k} + \mu(\mathbf{i} + \mathbf{j} + 4\mathbf{k})$		
			$1 + 2\lambda = 2 + \mu$		M1 (attempt to equate components, one correct)
			$3 + 3\lambda = 3 + \mu$		A1 (other correct) M1 (correct attempt to solve)
			$\lambda = -1,\ \mu = -3$		A1 (F.T. candidate's equations)
			Position vector of point of intersection is $-\mathbf{i} - 5\mathbf{k}$		A1 (C.A.O.)
	(b)	$\cos\theta$	$= \dfrac{(\mathbf{i} + 2\mathbf{j} - \mathbf{k}).(3\mathbf{i} - \mathbf{j} + 2\mathbf{k})}{\lvert\mathbf{i} + 2\mathbf{j} - \mathbf{k}\rvert\lvert 3\mathbf{i} - \mathbf{j} + 2\mathbf{k}\rvert}$		M1 (attempt to use correct formula)
			$= \dfrac{1\times 3 - 2\times 1 - 1\times 2}{\sqrt{1^2 + 2^2 + (-1)^2}\sqrt{3^2 + 1^2 + 2^2}}$		M1 (correct attempt to find scalar product) A1 (scalar product)
			$= \dfrac{-1}{\sqrt{6}\sqrt{14}}$		B1 (one correct modulus)
					B1 (F.T. arithmetic slip in scalar product)
		0	= 96·3° (accept nearest degree)		B1 (C.A.O.)
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10.	$3n + 2n^{3\text{-}}$	$= 3(2k) + 2(2k)^3$	
		$= 6k + 16k^3$	
		$= 2(3k + 8k^3)$	B1 (either $2x$ ( ) <u>or</u> even + even = even)
		which is even	B1
		Contradiction	
		(Thus n is odd)	
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