C2

Solutions and Mark Scheme

Final Version

1.

1	1·414213562		
1·1	1·337908816		
1·2	1·2489996		
1·3	1·144552314	(5 values correct)	B2
1·4	1·019803903	(3 or 4 values correct)	B1

Correct formula with $h = 0 \cdot 1$ M1

$I \approx \frac{0 \cdot 1}{2} \times \{1 \cdot 414213562 + 1 \cdot 019803903 + 2(1 \cdot 337908816 + 1 \cdot 2489996 + 1 \cdot 144552314)\}$

$I \approx 0 \cdot 494846946$

$I \approx 0 \cdot 495$ (f.t. one slip) A1

Special case for candidates who put $h = 0 \cdot 8$

1	1·414213562		
1·08	1·35410487		
1·16	1·286234815		
1·24	1·209297317		
1·32	1·121427662		
1·4	1·019803903	(all values correct)	B1

Correct formula with $h = 0 \cdot 08$ M1

$I \approx \frac{0 \cdot 08}{2} \times \{1 \cdot 414213562 + 1 \cdot 019803903 + 2(1 \cdot 35410487 + 1 \cdot 286234815 + 1 \cdot 209297317 + 1 \cdot 121427662)\}$

$I \approx 0 \cdot 495045871$

$I \approx 0 \cdot 495$ (f.t. one slip) A1

2. (*a*) $3 - 7\cos\theta = 6(1 - \cos^2\theta)$ (correct use of $\sin^2\theta = 1 - \cos^2\theta$) M1

An attempt to collect terms, form and solve quadratic equation in $\cos\theta$, either by using the quadratic formula or by getting the expression into the form $(a\cos\theta + b)(c\cos\theta + d)$, with $a \times c$ = coefficient of $\cos^2\theta$ and $b \times d$ = constant m1

$6\cos^2\theta - 7\cos\theta - 3 = 0 \Rightarrow (3\cos\theta + 1)(2\cos\theta - 3) = 0$

$\Rightarrow \cos\theta = -\frac{1}{3}$, ($\cos\theta = {}^3/_2$) (c.a.o.) A1

$\theta = 109{\cdot}47°, 250{\cdot}53°$ B1 B1

Note: Subtract (from final two marks) 1 mark for each additional root in range from $3\cos\theta + 1 = 0$, ignore roots outside range.
$\cos\theta = -$, f.t. for 2 marks, $\cos\theta = +$, f.t. for 1 mark

(*b*) $2x + 45° = 35°, 215°, 395°$ (one value) B1

$x = 85°, 175°$ B1, B1

Note: Subtract (from final two marks) 1 mark for each additional root in range, ignore roots outside range.

(*c*) Correct use of $\tan\theta = \frac{\sin\theta}{\cos\theta}$ (o.e.) M1

$\theta = 194{\cdot}48°$ A1

$\theta = 345{\cdot}52°$ A1

Note: Subtract (from final two marks) 1 mark for each additional root in range, ignore roots outside range.

3. (*a*) $x^2 = 8^2 + (x + 2)^2 - 2 \times 8 \times (x + 2) \times \cos 60°$

(correct use of cos rule) M1

$x^2 = 64 + x^2 + 4x + 4 - 8x - 16$ A1

$x = 13$ (f.t. only $x = 21$ from + 16 in the line above) A1

(*b*) $\frac{\sin ACB}{8} = \frac{\sin 60°}{13}$ (substituting correct values in the correct places in the sin rule, f.t. candidate's derived value for x) M1

$ACB = 32{\cdot}2°$ (f.t. candidate's derived value for x) A1

4. (*a*) At least one correct use of the sum formula M1

$\frac{8}{2} \times [2a + 7d] = 124$

$\frac{20}{2} \times [2a + 19d] = 910$ (both correct) A1

An attempt to solve the candidate's two equations simultaneously by eliminating one unknown M1

$d = 5$ (c.a.o.) A1

$a = -2$ (f.t. candidate's value for d) A1

(*b*) $-2 + 5(n - 1) = 183$ (f.t. candidate's values for a and d) M1

$n = 38$ (c.a.o.) A1

5. (*a*)

$S_n = a + ar + \ldots + ar^{n-1}$ (at least 3 terms, one at each end) B1

$rS_n = \quad ar + \ldots + ar^{n-1} + ar^n$

$S_n - rS_n = a - ar^n$ (multiply first line by r and subtract) M1

$(1 - r)S_n = a(1 - r^n)$

$S_n = \dfrac{a(1 - r^n)}{1 - r}$ (convincing) A1

(*b*)

Either: $\dfrac{a(1 - r^4)}{1 - r} = 73{\cdot}8$

Or: $a + ar + ar^2 + ar^3 = 73{\cdot}8$ B1

$\dfrac{a}{1 - r} = 125$ B1

An attempt to solve these equations simultaneously by eliminating one of the variables M1

$r^4 = 0{\cdot}4096$ A1

$r = 0{\cdot}8$ (c.a.o.) A1

$a = 25$ (f.t. candidate's value for r) A1

6. (*a*) $\frac{x^{4/3}}{4/3} - 2 \times \frac{x^{3/4}}{3/4} + c$ B1, B1

(–1 if no constant term present)

(*b*) (i) $5 + 4x - x^2 = 8$ M1

An attempt to rewrite and solve quadratic equation in x, either by using the quadratic formula or by getting the expression into the form $(x + a)(x + b)$, with $a \times b = 3$ m1

$(x - 1)(x - 3) = 0 \Rightarrow x = 1, 3$ (c.a.o.) A1

Note: Answer only with no working earns 0 marks

(ii) **Either:**

Total area $= \int_1^3 (5 + 4x - x^2)\,dx - \int_1^3 8\,dx$

(use of integration) M1

Either: $\int 5\,dx = 5x$ **and** $\int 8\,dx = 8x$ or: $\int 3\,dx = 3x$ B1

$\int 4x\,dx = 2x^2$, $\int x^2\,dx = \frac{x^3}{3}$ B1 B1

Total area $= [-3x + 2x^2 - (1/3)x^3]_1^3$ (o.e)

$= (-9 + 18 - 9) - (-3 + 2 - 1/3)$

(substitution of candidate's limits in at least one integral) m1

Subtraction of integrals with correct use of candidate's x_A, x_B as limits m1

Total area $= \frac{4}{3}$ (c.a.o.) A1

Or:

Area of rectangle = 16

(f.t. candidate's x-coordinates for A, B) B1

Area under curve $= \int_1^3 (5 + 4x - x^2)\,dx$

(use of integration) M1

$= [5x + 2x^2 - (1/3)x^3]_1^3$

(correct integration) B2

$= (15 + 18 - 9) - (5 + 2 - 1/3)$

(substitution of candidate's limits) m1

$= \frac{52}{3}$

Use of candidate's, x_A, x_B as limits and trying to find total area by subtracting area of rectangle from area under curve m1

Total area $= \frac{52}{3} - 16 = \frac{4}{3}$ (c.a.o.) A1

7. (*a*) Let $p = \log_a x$

Then $x = a^p$ (relationship between log and power) B1

$x^n = a^{pn}$ (the laws of indices) B1

$\therefore \log_a x^n = pn$ (relationship between log and power)

$\therefore \log_a x^n = pn = n \log_a x$ (convincing) B1

(*b*) $\frac{1}{2} \log_a 324 = \log_a 324^{1/2}$

$2 \log_a 12 = \log_a 12^2$ (at least one use of power law) B1

$\frac{1}{2} \log_a 324 + \log_a 56 - 2 \log_a 12 = \log_a \frac{324^{1/2} \times 56}{12^2}$

(use of addition law) B1

(use of subtraction law) B1

$\frac{1}{2} \log_a 324 + \log_a 56 - 2 \log_a 12 = \log_a 7$ (c.a.o) B1

Note: Answer only of $\log_a 7$ without any working earns 0 marks

(*c*) (i) $2^{x+1} = 2^x \times 2$ B1

$3^x = 2^{x+1} \Rightarrow (1\cdot5)^x = 2$ B1

(ii) **Hence:** $x \log_{10} 1\cdot5 = \log_{10} 2$

(taking logs on both sides and using the power law) M1

(f.t. candidate's values for *c* and *d*)

$x = 1\cdot71$ (c.a.o.) A1

Otherwise:

$x \log_{10} 3 = (x + 1) \log_{10} 2$

(taking logs on both sides and using the power law) M1

$x = 1\cdot71$ (c.a.o.) A1

8. (*a*) $A(-2, 4)$ B1

A correct method for finding radius M1

Radius = $\sqrt{10}$ A1

(*b*) An attempt to substitute $(3y - 4)$ for x in the equation of the circle M1

$10y^2 - 20y + 10 = 0$ A1

Either: Use of $b^2 - 4ac$ m1

Determinant = 0 $\Rightarrow x - 3y + 4 = 0$ is a tangent to the circle A1

Or: An attempt to factorise candidate's quadratic m1

Repeated (single) root $\Rightarrow x - 3y + 4 = 0$ is a tangent to the circle A1

9. (*a*) $\frac{1}{2} \times 6^2 \times \sin\theta = 9{\cdot}1$ M1

$\theta = 0{\cdot}53$ A1

(*b*) Substitution of values in formula for area of sector M1

Area = $^1/_2 \times 6^2 \times 0{\cdot}53 = 9{\cdot}54$ cm^2 (f.t. candidate's value for θ) A1

(*c*) $6 + 6 + 6\varphi = \pi \times 6$ M1

$\varphi = 1{\cdot}14$ A1

10. (*a*) $t_3 = 31$ B1

$t_1 = 7$ (f.t. candidate's value for t_3) B1

(*b*) All terms of the sequence are odd numbers E1