Mathematics C2 May 2008

Solutions and Mark Scheme

1.

0	1		
0·2	1·060596059		
0·4	1·249358235	(2 values correct)	B1
0·6	1·586018915	(4 values correct)	B1

Correct formula with $h = 0{\cdot}2$ M1

$I \approx \frac{0{\cdot}2}{2} \times \{1 + 1{\cdot}586018915 + 2(1{\cdot}060596059 + 1{\cdot}249358235)\}$

$I \approx 0{\cdot}72059275$

$I \approx 0{\cdot}721$ (f.t. one slip) A1

Special case for candidates who put $h = 0{\cdot}15$

0	1		
0·15	1·033939138		
0·3	1·137993409		
0·45	1·318644196		
0·6	1·586018915	(all values correct)	B1

Correct formula with $h = 0{\cdot}15$ M1

$I \approx \frac{0{\cdot}15}{2} \times \{1 + 1{\cdot}586018915 + 2(1{\cdot}033939138 + 1{\cdot}137993409 + 1{\cdot}318644196)\}$

$I \approx 0{\cdot}71753793$

$I \approx 0{\cdot}718$ (f.t. one slip) A1

2. (*a*) $\tan\theta = 1{\cdot}5$ (c.a.o.) B1

$\theta = 56{\cdot}31°$ B1

$\theta = 236{\cdot}31°$ B1

Note: Subtract (from final two marks) 1 mark for each additional root in range, ignore roots outside range.

(*b*) $3x = 25{\cdot}84°, 334{\cdot}16°, 385{\cdot}84°, 694{\cdot}16°$ (one value) B1

$x = 8{\cdot}61°$ (f.t candidate's value for $3x$) B1

$x = 111{\cdot}39°, 128{\cdot}61°$ (f.t candidate's value for $3x$) B1, B1

Note: Subtract (from final two marks) 1 mark for each additional root in range, ignore roots outside range.

(*c*) $\sin^2\theta - 4(1 - \sin^2\theta) = 8\sin\theta$

(correct use of $\cos^2\theta = 1 - \sin^2\theta$) M1

An attempt to collect terms, form and solve quadratic equation in $\sin\theta$, either by using the quadratic formula or by getting the expression into the form $(a\sin\theta + b)(c\sin\theta + d)$, with $a \times c$ = coefficient of $\sin^2\theta$ and $b \times d$ = constant m1

$5\sin^2\theta - 8\sin\theta - 4 = 0 \Rightarrow (5\sin\theta + 2)(\sin\theta - 2) = 0$

$\Rightarrow \sin\theta = -\frac{2}{5}$, ($\sin\theta = 2$) (c.a.o.) A1

$\theta = 203{\cdot}58°, 336{\cdot}42°$ B1 B1

Note: Subtract (from final two marks) 1 mark for each additional root in range from $5\sin\theta + 2 = 0$, ignore roots outside range.

$\sin\theta = -$, f.t. for 2 marks, $\sin\theta = +$, f.t. for 1 mark

3. (*a*) $15 = \frac{1}{2} \times x \times (x + 4) \times \sin 150°$ (correct use of area formula) M1

Either $x(x + 4) = 60$ or expressing the equation correctly in the form $ax^2 + bx + c = 0$ A1

$x = 6$ (negative value must be rejected) (c.a.o.) A1

(*b*) $BC^2 = 6^2 + 10^2 - 2 \times 6 \times 10 \times \cos 150°$

(correct substitution of candidate's derived values in cos rule) M1

$BC = 15{\cdot}5$ cm (f.t. candidate's derived value for x) A1

4. (*a*) $S_n = a + [a + d] + \ldots + [a + (n-2)d] + [a + (n-1)d]$

(at least 3 terms, one at each end) B1

$S_n = [a + (n-1)d] + [a + (n-2)d] + \ldots + [a + d] + a$

$2S_n = [a + a + (n-1)d] + [a + a + (n-1)d] + \ldots + [a + a + (n-1)d]$

(reverse and add) M1

$2S_n = n[2a + (n-1)d]$

$S_n = \frac{n}{2}[2a + (n-1)d]$ (convincing) A1

(*b*) $\frac{10}{2} \times [2a + 9d] = 320$ B1

$2a + 9d = 64$

$[a + 11(12)d] + [a + 15(16)d] = 166$ M1

$[a + 11d] + [a + 15d] = 166$ A1

$2a + 26d = 166$

An attempt to solve the candidate's two equations simultaneously by eliminating one unknown M1

$d = 6$, $a = 5$ (both values) (c.a.o.) A1

5. $a + ar = 7{\cdot}2$ B1

$\frac{a}{1 - r} = 20$ B1

A valid attempt to eliminate a M1

$20(1 - r) + 20(1 - r)r = 7{\cdot}2$ (a correct quadratic in r) A1

$r = 0{\cdot}8$ ($r = -0{\cdot}8$) (c.a.o.) A1

$r = 0{\cdot}8$ and $a = 4$ (f.t. candidate's positive value for r) A1

6. (*a*) $5 \times \frac{x^{3/2}}{3/2} - 4 \times \frac{x^{1/3}}{1/3}$ $(+c)$ B1,B1

(*b*) (i) $4 - x^2 = 3x$ M1

An attempt to rewrite and solve quadratic equation in x, either by using the quadratic formula or by getting the expression into the form $(x + a)(x + b)$, with $a \times b = -4$ m1

$(x - 1)(x + 4) = 0 \Rightarrow x = 1 \quad (-4)$ (c.a.o.) A1

$A(1, 3)$ (f.t. candidate's x-value, dependent on M1 only) A1

$B(2, 0)$ B1

(ii) Area of triangle = 3/2

(f.t. candidate's coordinates for A) B1

Area under curve $= \int_1^2 (4 - x^2)\, dx$

(use of integration) M1

$= [4x - (1/3)x^3]_1^2$

(correct integration) B2

$= [(8 - 8/3) - (4 - 1/3)]$

(substitution of candidate's limits) m1

$= 5/3$

Use of candidate's, x_A, x_B as limits and trying to find total area by adding area of triangle and area under curve m1

Total area = 3/2 + 5/3 = 19/6 (c.a.o.) A1

7. (*a*) Let $p = \log_a x$

Then $x = a^p$ (relationship between log and power) B1

$x^n = a^{pn}$ (the laws of indicies) B1

$\therefore \log_a x^n = pn$ (relationship between log and power)

$\therefore \log_a x^n = pn = n\log_a x$ (convincing) B1

(*b*) $\log_a(3x + 4) - \log_a x = 3\log_a 2$.

$\log_a \left(\frac{3x+4}{x}\right) = \log_a 2^3$ (use of subtraction law) B1

(use of power law) B1

$\frac{3x+4}{x} = 2^3$ (removing logs) (c.a.o) B1

$x = 0{\cdot}8$ (f.t. for $2^3 = 6, 9$ only) B1

(*c*) **Either:**

$(3y + 2)\log_{10} 4 = \log_{10} 7$

(taking logs on both sides and using the power law) M1

$y = \frac{\log_{10} 7 \pm 2\log_{10} 4}{3\log_{10} 4}$ m1

$y = -0{\cdot}199$ (c.a.o.) A1

Or:

$3y + 2 = \log_4 7$ (rewriting as a log equation) M1

$y = \frac{\log_4 7 \pm 2}{3}$ m1

$y = -0{\cdot}199$ (c.a.o.) A1

8. (*a*) (i) $A(5, 3)$ B1

(ii) A correct method for finding radius M1

Radius = $\sqrt{65}$ A1

(iii) Equation of C: $(x - 5)^2 + (y - 3)^2 = (\sqrt{65})^2$

(f.t. candidate's coordinates of A.) B1

(*b*) **Either:**

An attempt to substitute the coordinates of R in the equation of C M1

Verification that L.H.S. of equation of $C = 65 \Rightarrow R$ lies on C A1

Or:

An attempt to find AR^2 M1

$AR^2 = 65 \Rightarrow R$ lies on C A1

(*c*) **Either:**

$RQ = \sqrt{26}$ ($RP = \sqrt{234}$) B1

$\sin QPR = \frac{\sqrt{26}}{2\sqrt{65}}$ $\left[\cos QPR = \frac{\sqrt{234}}{2\sqrt{65}}\right]$ M1

$QPR = 18{\cdot}43°$ A1

Or:

$RQ = \sqrt{26}$ or $RP = \sqrt{234}$ B1

$(\sqrt{26})^2 = (\sqrt{234})^2 + (2\sqrt{65})^2 - 2 \times (\sqrt{234}) \times (2\sqrt{65}) \times \cos QPR$

(correct use of cos rule) M1

$QPR = 18{\cdot}43°$ A1

9. Let $A\hat{O}B = \theta$ radians

(*a*) $6 \times \theta = 5{\cdot}4$ M1

$\theta = 0{\cdot}9$ A1

Area of sector $AOB = \frac{1}{2} \times 6^2 \times \theta$ M1

Area of sector $AOB = 16{\cdot}2\ \text{cm}^2$ (convincing) A1

(*b*) Area of triangle $AOB = \frac{1}{2} \times 6^2 \times \sin\theta$ M1

Area of triangle $AOB = 14{\cdot}1\ \text{cm}^2$ (f.t. candidate's value for θ) A1

Shaded area = $2{\cdot}1\ \text{cm}^2$ (f.t. candidate's value for θ) A1