MATHEMATICS C2

1.

0	1		
$\pi/8$	1·175875602		
$\pi/4$	1·306562965		
$3\pi/8$	1·387039845	(3 values correct)	B1
$\pi/2$	1·414213562	(5 values correct)	B1

Correct formula with $h = \pi/8$ M1

$I \approx \frac{\pi/8}{2} \times \{1 + 1·414213562 + 2(1·175875602 + 1·306562965 + 1·387039845)\}$

$I \approx 1·994$ (f.t. one slip) A1

Special case for candidates who put $h = \pi/10$

0	1		
$\pi/10$	1·144122806		
$\pi/5$	1·260073511		
$3\pi/10$	1·344997024		
$2\pi/5$	1·396802247		
$\pi/2$	1·414213562	(all values correct)	B1

Correct formula with $h = \pi/10$ M1

$I \approx \frac{\pi/10}{2} \times \{1 + 1·414213562 + 2(1·144122806 + 1·260073511 + 1·344997024 + 1·396802247)\}$

$I \approx 1·9$ (f.t. one slip) A1

2. (*a*) $3x = 60°, 240°, 420°, 600°$ (one value) B1

$x = 20°, 80°, 140°$ B1, B1, B1

Note: Subtract 1 mark for each additional root in range, ignore roots outside range.

(*b*) $4\cos^2\theta - \cos\theta = 2(1 - \cos^2\theta)$ (correct use of $\sin^2\theta = 1 - \cos^2\theta$) M1

An attempt to collect terms, form and solve quadratic equation in $\cos\theta$, either by using the quadratic formula or by getting the expression into the form $(a\cos\theta + b)(c\cos\theta + d)$, with $a \times c$ = coefficient of $\cos^2\theta$ and $b \times d$ = constant m1

$6\cos^2\theta - \cos\theta - 2 = 0 \Rightarrow (3\cos\theta - 2)(2\cos\theta + 1) = 0$

$\Rightarrow \cos\theta = \frac{2}{3}, -\frac{1}{2}$ A1

$\theta = 48·19°, 311·81°, 120°, 240°$ (48·19°, 311·81°) B1

(120°) B1

(240°) B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.

$\cos\theta = +, -$, f.t. for 3 marks, $\cos\theta = -, -$, f.t. for 2 marks

$\cos\theta = +, +$, f.t. for 1 mark

3. (*a*) $7^2 = x^2 + (3x)^2 - 2 \times x \times 3x \times \cos 60°$ (correct use of cos rule) M1
$7^2 = x^2 + 9x^2 - 3x^2$ A1
$7x^2 = 7^2 \Rightarrow x = \sqrt{7}$ (convincing) A1

(*b*) **Either:**
$\frac{7}{\sin 60°} = \frac{\sqrt{7}}{\sin ACB}$ (correct use of sin rule) M1
$ACB = 19·11°$ A1

Or:
$(\sqrt{7})^2 = 7^2 + (3\sqrt{7})^2 - 2 \times 7 \times (3\sqrt{7}) \times \cos ACB$
(correct use of cos rule) M1
$ACB = 19·11°$ A1

4. (*a*) $a + 2d = k(a + 5d)$ ($k = 4, \frac{1}{4}$) M1
$a + 2d = 4(a + 5d)$ A1
$\frac{20}{2}[2a + 19d] = 350$ B1
An attempt to solve simultaneous equations M1
$d = 5$ ($a = -30$) (c.a.o.) A1
$a = -30$ ($d = 5$) (f.t. one error) A1

(*b*) $-30 + (n - 1) \times 5 = 125$ (equation for n'th term and an attempt to solve, f.t. candidate's values for a, d) M1
$n = 32$ (c.a.o.) A1

5. (*a*) $S_n = a + ar + \ldots + ar^{n-2} + ar^{n-1}$ (at least 3 terms, one at each end) B1
$rS_n = ar + ar^2 + \ldots ar^{n-1} + ar^n$
$S_n - rS_n = a - ar^n$ (multiply first line by r and subtract) M1
$(1 - r)S_n = a(1 - r^n)$
$S_n = \frac{a(1 - r^n)}{1 - r}$ (convincing) A1
$S_\infty = \frac{a}{1 - r}$ B1

(*b*) (i) $\frac{a}{1 - r} = 10$ B1
$\frac{a}{1 - 2r} = 15$ B1
An attempt to eliminate a M1
$r = 0·25$ (c.a.o.) A1

(ii) $a = 7·5$ B1
$S_4 = \frac{7·5[1 - 0·25^4]}{1 - 0·25}$
(award even if sum calculated for 2nd series) M1
$S_4 \approx 9·96$ (f.t. candidate's derived values of a, r) A1

6. (*a*) $\frac{2x^{5/2}}{5/2} + \frac{9x^{-3}}{-3}$ $(+c)$ B1,B1

(*b*) (i) $x^2 + 2 = 3x$ M1

An attempt to rewrite and solve quadratic equation in x, either by using the quadratic formula or by getting the expression into the form $(x + a)(x + b)$, with $a \times b = 2$ m1

$(x - 2)(x - 1) = 0 \Rightarrow x = 1, x = 2$ (both values, c.a.o.) A1

$A(1, 3)$, $B(2, 6)$ (both values, f.t. candidate's x values) A1

(ii)

Either:

Total area $= \int_0^1 3x\,dx + \int_1^2 (x^2 + 2)\,dx$ (use of integration) M1

(addition of integrals) m1

$= [(3/2)x^2]_0^1 + [(1/3)x^3 + 2x]_1^2$ (correct integration) B3

$= [3/2 - 0] + [(8/3 + 4) - (1/3 + 2)]$

(use of candidate's $0, x_A, x_A, x_B$ as limits) M1

$= 35/6$ (c.a.o.) A1

Or:

Area of triangle = 3/2 (f.t. candidate's coordinates for A) B1

Area under curve $= \int_1^2 (x^2 + 2)\,dx$ (use of integration) M1

$= [(1/3)x^3 + 2x]_1^2$ (correct integration) B2

$= (8/3 + 4) - (1/3 + 2)$

(use of candidate's x_A, x_B as limits) M1

$= 13/3$

Finding total area by adding values m1

Total area = 3/2 + 13/3 = 35/6 (c.a.o.) A1

7. (*a*) (i) Let $x = \log_a p$, $y = \log_a q$

Then $p = a^x$, $q = a^y$ (relationship between log and power) B1

$pq = a^x \times a^y = a^{x+y}$ (the laws of indicies) B1

$\log_a pq = x + y = \log_a p + \log_a q$ (convincing) B1

(ii) $\log_a x + \log_a(3x + 4) = \log_a x(3x + 4)$ (addition law) B1

$2\log_a(3x - 4) = \log_a(3x - 4)^2$ (power law) B1

$x(3x + 4) = (3x - 4)^2$ (removing logs) M1

An attempt to rearrange and solve quadratic m1

$3x^2 - 14x + 8 = 0 \Rightarrow x = 2/3, x = 4$ (c.a.o.) A1

(*b*) **Either:**

$x\log_{10} 3 = \log_{10} 11$ B1

$x = \dfrac{\log_{10} 11}{\log_{10} 3} \Rightarrow x \approx 2{\cdot}183$ B1

Or:

$x = \log_3 11$ B1

$x \approx 2{\cdot}183$ B1

8. (*a*) $A(-2, 8)$ B1

A correct method for finding the radius M1

Radius = $\sqrt{50}$ A1

(*b*) An attempt to substitute $(x + 2)$ for y in the equation of the circle M1

$x^2 - 4x - 5 = 0$ (or $2x^2 - 8x - 10 = 0$) B1

$x = -1, x = 5$ (correctly solving candidate's quadratic, both values) A1

Points of intersection are $(-1, 1), (5, 7)$ (c.a.o.) A1

9. (*a*) Length of arc = 6θ B1

Circumference of circle = $2 \times \pi \times 6$

B1

$2 \times \pi \times 6 = 2 \times 6\theta + 24$

(expression of the given information as an equation using the candidate's expressions for length of arc and circumference) M1

$\theta = \pi - 2$ (convincing) A1

(*b*) **Either:**

Area of unshaded sector = $(1/2) \times 6^2 \times (\pi - 2)$ [= 20·55] B1

Area of shaded sector = $36\pi - (1/2) \times 6^2 \times (\pi - 2)$

(f.t. candidate's expression/value for area of unshaded sector) M1

Area of shaded sector = 92·55 cm^2 (c.a.o.) A1

Or:

Angle in shaded sector = $2\pi - (\pi - 2)$ [= 5·14] B1

Area of shaded sector = $(1/2) \times 6^2 \times [2\pi - (\pi - 2)]$ M1

Area of shaded sector = 92·52 cm^2 (c.a.o.) A1

Accept answers in the interval [92·5, 92·6]