MATHEMATICS C2

1. *(a)* $h = 0.2$ — M1 (correct formula $h = 0.2$)

Integral $\approx \frac{0.2}{2}$ [0.5 + 0.3333333 + 2(0.4980080 + 0.48449612 — B1 (4 values)

+ 0.4512635 + 0.3980892)] — B1 (2 values)

$\approx$ 0.4497 — A1 (F.T. one slip)

S. Case $h = \frac{1}{6}$ — M1 (correct formula, $h = \frac{1}{6}$)

Integral $\approx \frac{1}{12}$ [0.5 + 0.3333333 + 2(0.4988453 + 0.4909091 — B1 (all values)

+ 0.4705882 + 0.4354838 + 0.3877917)]

$\approx$ 0.4500 — A1 (F.T. one slip)

4

2. *(a)* $4\cos^2\theta - \cos\theta = 2(1 - \cos^2\theta)$ — M1 (correct use of $\sin^2\theta + \cos^2\theta = 1$)

$6\cos^2\theta - \cos\theta - 2 = 0$ — M1 (attempt to solve quadratic in $\cos\theta$, correct formula)

$(3\cos\theta - 2)(2\cos\theta + 1) = 0$

$\cos\theta = \frac{2}{3}, -\frac{1}{2}$ — A1 (C.A.O.)

$\theta = 48.2^\circ, 311.8^\circ, 120^\circ, 240^\circ$ — B1 (48.2, 311.8°) B1 (120°) B1 (240°)

(b) $\tan\theta = -\sqrt{3}$

$\theta = 120^\circ, 300^\circ$ — B1, B1

(c) $2\theta = 30^\circ, 150^\circ, 390^\circ, 510^\circ$ — B1 (one value)

$\theta = 15^\circ, 75^\circ, 195^\circ, 255^\circ$ — B1 (2 values) B1 (2 values)

11

3.

(a) Sine rule $\frac{10}{\sin 45°} = \frac{12}{\sin A\hat{C}B}$ — M1 (correct use of sine rule)

$\sin A\hat{C}B = \frac{12 \sin 45°}{10}$

$A\hat{C}B$ = 58.05º or 121.95º — A1 (one value)

$A\hat{B}C$ = 180º – (45º + 58.05º) = 76.95º — A1 (F.T. one slip)

or 180° – (45º + 121.95º) = 13.05º — A1

(b) Area $= \frac{1}{2} \times 12 \times 10 \sin 76.95º \approx 58.4$ cm^2 (58.4 – 58.5) — M1 (correct formula)

or $\frac{1}{2} \times 12 \times 10 \sin 13.05º \approx 13.4$ cm^2 — A1 (both)

(F.T. candidate values)

6

4. *(a)* nth term = ar^{n-1} — B1

Let $S_n = a + ar + ar^2 + ... + ar^{n-2} + ar^{n-1}$ (1) — B1 (at least 3 terms, one at each end)

$rS_n = ar + ar^2 + ... + ar^{n-2} + ar^{n-1} + ar^n$ (2) — M1 (multiplication by r and subtract)

(1) – (2)

$S_n - rS_n = a - ar^n$

$S_n (1 - r) = a (1 - r^n)$

$S_n = \frac{a(1 - r^n)}{1 - r}$ — A1 (convincing)

(b) (i) $ar^3 = 2,\ ar^6 = 54$ — B1 (both)

$r^3 = 27$ — M1 (eliminate a)

$r = 3$ — A1

(ii) $a = \dfrac{2}{3^3} = \dfrac{2}{27}$ — B1 (F.T. one slip)

$S_{10} = \dfrac{2}{27}\dfrac{(1-3^{10})}{1-3} \approx 2187.0$ — M1 (correct formula, F.T. candidate values)

A1

(iii) $\dfrac{2}{27}3^{n-1} = 125000$ — B1 (F.T. candidate values)

$\therefore\ 3^{n-1} = 1687500$

$(n-1)\ln 3 = \ln 1687500$ — M1 (attempt to take logs)

$n = 1 + 13.05 = 14.05$ — A1 (C.A.O.)

Least value is 15 — A1 (F.T. candidate's n)

14

5. *(a)* $2a + d = 3$ (1) — B1 ($a + a + d = 3$)

$a + 7d = 47$ (2) — B1

Solve (1), (2) $d = 7,\ a = -2$ — M1 (attempt to solve)

A1 (C.A.O.)

(b) $S_{20} = \dfrac{20}{2}\ [2 \times -2 + 19 \times 7]$ — M1 (correct formula with candidate values)

$= 1290$ — A1 (F.T. candidate values)

6

6. $\dfrac{5x^{\frac{4}{3}}}{\frac{4}{3}} + \dfrac{3x^{-2}}{-2}$ (+ C) — B1, B1

$\left(\dfrac{15}{4}x^{\frac{4}{3}} - \dfrac{3}{2x^2}\right)$

2

7. *(a)* $\underline{B}$ $y = 0,\ 4 - x^2 = 0$ — M1 (setting $y = 0$)

$x = \pm 2$

$B\ (2, 0)$ — A1

$\underline{A}$ $4 - x^2 = 3x$ — M1 (equating ys)

$x^2 + 3x - 4 = 0$ — M1 (correct attempt to solve quad)

$x = -4,\ 1$

$A(1, 3)$ — A1

(b) Area $= \int_0^1 3x\,dx + \int_1^2 (4 - x^2)\,dx$

M1 (use of integration to find area)

M1 (addition of areas)

$= \left[\frac{3x^2}{2}\right]_0^1 + \left[4x - \frac{x^3}{3}\right]_1^2$

B3 (integration)

$= \frac{3}{2} + 8 - \frac{8}{3} - 4 + \frac{1}{3}$

M1 (use of candidate's limits, any order)

$= \frac{19}{6}$

A1 (C.A.O.)

12

8. *(a)* Centre (4, –2)

B1

Radius $= \sqrt{4^2 + (-2)^2 - 1} = 3$

M1 (correct method of finding radius)
A1

(b) Centre is (0, 0), radius = a

B1 (both)

Distance between centres = $\sqrt{4^2 + 2^2} = \sqrt{20}$

B1

Circles touch if

$\sqrt{20} = a + 3$

M1 (F.T. distance)

$a \approx 1.47$

A1 (F.T. distance)

7

9. *(a)* $\frac{1}{2} 4^2 (\theta + \phi) = 15.2$

M1 (use of correct formula)

$\theta + \phi = \frac{15.2}{8} = 1.9 \qquad (1)$

A1 (convincing)

(b) $4\theta - 4\phi = 3.2$

M1 (use of correct formula)

$\theta - \phi = 0.8 \qquad (2)$

A1

Solve (1), (2)

$\theta = 1.35, \phi = 0.55$

M1 (attempt to solve)
A1 (F.T. one slip)

6

10. *(a)* Let $x = a^p$, $y = a^q$ — B1 (properties of $\log_a x$ and $x = a^p$)

$\log_a x = p$, $\log_a y = q$

$xy = a^{p+q} = a^{p+q}$ — B1 (laws of indices)

$\log_a (xy) = p + q = \log_a x + \log_a y$ — B1 (convincing)

(b) $\int_1^3 \log_{10} x dx + \int_1^3 \log_{10} 10 dx$ — B1 laws of logs)

$\approx 0.5628 + \int_1^3 1 dx$ — B1

$= 0.5628 + \left[x\right]_1^3$, — B1 (integration)

$= 0.5628 + 3 - 1$ — B1

$= 2.5628$

7