Mathematics C1 January 2010

Solutions and Mark Scheme

Final Version

1. (*a*) Gradient of $BC = \dfrac{\text{increase in } y}{\text{increase in } x}$ M1

Gradient of $BC = -1/2$ (or equivalent) A1

(*b*) (i) Use of gradient L_1 = gradient BC M1

A correct method for finding the equation of L_1 using candidate's gradient for L_1 M1

Equation of L_1: $y - 10 = -1/2\,[x - (-11)]$ (or equivalent)

(f.t. candidate's gradient for BC) A1

Equation of L_1: $x + 2y - 9 = 0$ (convincing) A1

(ii) Use of gradient $L_2 \times$ gradient $BC = -1$ M1

A correct method for finding the equation of L_2 using candidate's gradient for L_2 M1

(to be awarded only if corresponding M1 is not awarded in part (b)(i))

Equation of L_2 : $y - 8 = 2(x - 3)$ (or equivalent)

(f.t. candidate's gradient for BC) A1

(*c*) (i) An attempt to solve equations of L_1 and L_2 simultaneously M1

$x = 1,\ y = 4$ (convincing.) A1

(ii) A correct method for finding the length of BD M1

$BD = 10$ A1

(iii) A correct method for finding the coordinates of the mid-point of BD M1

Mid-point of BD has coordinates $(-2, 8)$ A1

2. (*a*) $\dfrac{2\sqrt{11}-3}{\sqrt{11}+2} = \dfrac{(2\sqrt{11}-3)(\sqrt{11}-2)}{(\sqrt{11}+2)(\sqrt{11}-2)}$ M1

Numerator: $22 - 4\sqrt{11} - 3\sqrt{11} + 6$ A1

Denominator: $11 - 4$ A1

$\dfrac{2\sqrt{11}-3}{\sqrt{11}+2} = 4 - \sqrt{11}$ (c.a.o.) A1

Special case

If M1 not gained, allow B1 for correctly simplified numerator or denominator following multiplication of top and bottom by $\sqrt{11}+2$

(*b*) $\dfrac{22}{\sqrt{2}} = 11\sqrt{2}$ B1

$\sqrt{50} = 5\sqrt{2}$ B1

$(\sqrt{2})^5 = 4\sqrt{2}$ B1

$\dfrac{22}{\sqrt{2}} - \sqrt{50} - (\sqrt{2})^5 = 2\sqrt{2}$ (c.a.o.) B1

3. An attempt to differentiate, at least one non-zero term correct M1

$\dfrac{dy}{dx} = 6 \times -2 \times x^{-3} + \dfrac{7}{4}$ A1

An attempt to substitute $x = 2$ in candidate's derived expression for $\dfrac{dy}{dx}$ M1

Value of $\dfrac{dy}{dx}$ at $P = \dfrac{1}{4}$ (c.a.o.) A1

Gradient of normal = $\dfrac{-1}{\text{candidate's derived value for } \frac{dy}{dx}}$ M1

Equation of normal to C at P: $y - 3 = -4(x - 2)$ (or equivalent)

(f.t. candidate's value for $\dfrac{dy}{dx}$ provided all three M1's are awarded) A1

4. (*a*) $a = 4$ B1

$b = -1$ B1

$c = 3$ B1

(*b*) $\dfrac{1}{c}$ on its own or greatest value = $\dfrac{1}{c}$,

with correct explanation or no explanation B2

If B2 not awarded

$\dfrac{1}{c}$ on its own or greatest value = $\dfrac{1}{c}$, with incorrect explanation B1

least value = $\dfrac{1}{c}$ with no explanation B1

least value = $\dfrac{1}{c}$ with incorrect explanation B0

5. (*a*) An expression for $b^2 - 4ac$, with at least two of a, b or c correct — M1

$b^2 - 4ac = 3^2 - 4 \times k \times (-5)$ — A1

$b^2 - 4ac < 0$ — m1

$k < -{}^9/_{20}$

(f.t. only for $k > {}^9/_{20}$ from $b^2 - 4ac = 3^2 - 4 \times k \times 5$) — A1

(*b*) Finding critical values $x = -1\cdot5$, $x = 2$ — B1

A statement (mathematical or otherwise) to the effect that

$x < -1\cdot5$ or $2 < x$ (or equivalent)

(f.t critical values $\pm 1\cdot5$, ± 2 only) — B2

Deduct 1 mark for each of the following errors

the use of $\leq$ rather than $<$

the use of the word 'and' instead of the word 'or'

6. (*a*) $y + \delta y = 3(x + \delta x)^2 - 7(x + \delta x) - 5$ — B1

Subtracting y from above to find δy — M1

$\delta y = 6x\delta x + 3(\delta x)^2 - 7\delta x$ — A1

Dividing by δx and letting $\delta x \to 0$ — M1

$$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x} = 6x - 7$$ (c.a.o.) A1

(*b*) $\frac{dy}{dx} = a \times \frac{5}{2} \times x^{3/2}$ — B1

Substituting $x = 4$ in candidate's expression for $\frac{dy}{dx}$ and putting expression equal to -2 — M1

$a = -\frac{1}{10}$ (c.a.o.) A1

7. Coefficient of $x = {}^5C_1 \times a^4 \times 3(x)$ — B1

Coefficient of $x^2 = {}^5C_2 \times a^3 \times 3^2(x^2)$ — B1

$10 \times a^3 \times m = k \times 5 \times a^4 \times 3$ (o.e.) ($m = 9$ or 3, $k = 8$ or ${}^1/_8$) — M1

$a = \frac{3}{4}$ (c.a.o.) A1

8. (*a*) $f(-2) = 15$ B1

Either: When $f(x)$ is divided by $x + 2$, the remainder is 15

Or: $x + 2$ is not a factor of $f(x)$

[f.t. candidate's value for $f(-2)$] E1

(*b*) Attempting to find $f(r) = 0$ for some value of r M1

$f(-1) = 0 \Rightarrow x + 1$ is a factor A1

$f(x) = (x + 1)(2x^2 + ax + b)$ with one of a, b correct M1

$f(x) = (x + 1)(2x^2 + 9x - 5)$ A1

$f(x) = (x + 1)(x + 5)(2x - 1)$ (f.t. only $2x^2 - 9x - 5$ in above line) A1

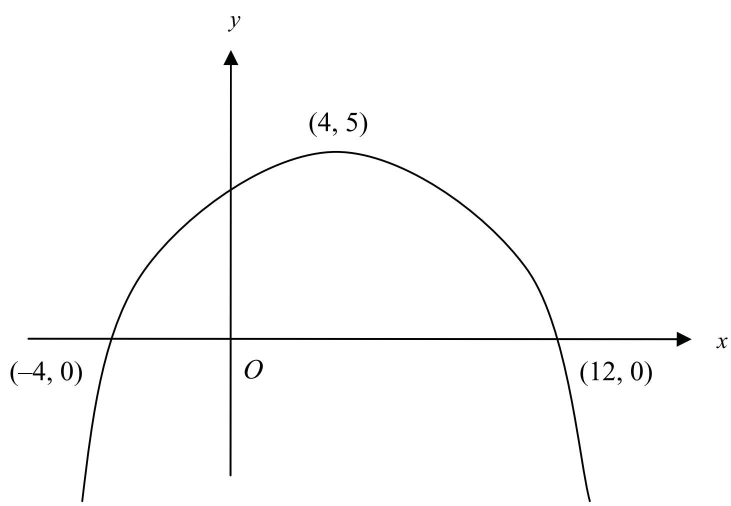
Roots are $x = -1, -5, 1/2$

(f.t. only from $(x + 1)(x - 5)(2x + 1)$ in above line) A1

Special case

Candidates who, after having found $x + 1$ as one factor, then find one of the remaining factors by using e.g. the factor theorem, are awarded B1

9. (*a*)



Concave down curve and y-coordinate of maximum = 5 B1

x-coordinate of maximum = 4 B1

Both points of intersection with x-axis B1

(*b*) $y = f(x - 4)$ B2

If B2 not awarded

$y = f(x + 4)$ B1

10. (*a*) $\frac{dy}{dx} = 3x^2 - 12x$ B1

Putting derived $\frac{dy}{dx} = 0$ M1

$x = 0, 4$ (both correct) (f.t. candidate's $\frac{dy}{dx}$) A1

Stationary points are (0, 20) and (4, –12) (both correct) (c.a.o) A1

A correct method for finding nature of stationary points yielding

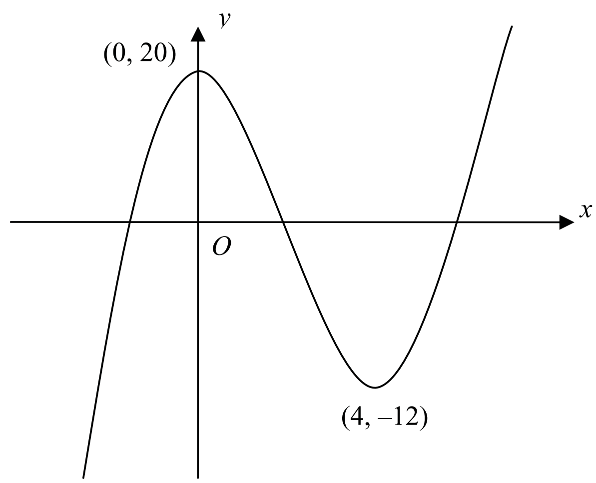
either (0, 20) is a maximum point

or (4, –12) is a minimum point (f.t. candidate's derived values) M1

Correct conclusion for other point

(f.t. candidate's derived values) A1

(*b*)



Graph in shape of a positive cubic with two turning points M1

Correct marking of both stationary points

(f.t. candidate's derived maximum and minimum points) A1

(*c*) Use of both $k = -12$, $k = 20$ to find the range of values for k

(f.t. candidate's y-values at stationary points) M1

$-12 < k < 20$ (f.t. candidate's y-values at stationary points) A1