Mathematics C1 May 2009

Solutions and Mark Scheme

1. (*a*) Gradient of $AB = \frac{\text{increase in } y}{\text{increase in } x}$ M1

Gradient of $BC = {}^{3}/_{4}$ (or equivalent) A1

(*b*) A correct method for finding C M1

C (3, 8) A1

(*c*) Use of $m_{AB} \times m_L = -1$ to find gradient of L M1

A correct method for finding the equation of L using candidate's coordinates for C and candidate's gradient for L. M1

Equation of L: $y - 8 = -{}^{4}/_{3}\,(x - 3)$ (or equivalent)
(f.t. candidate's coordinates for C and candidate's gradient for L) A1

Equation of L: $4x + 3y - 36 = 0$ (convincing, c.a.o.) A1

(*d*) (i) Substituting $y = 0$ in equation of L M1

D (9, 0) A1

(ii) A correct method for finding the length of CD (AC) M1

$CD = 10$ (f.t. candidate's coordinates for C and D) A1

(iii) $AC = 5$ (f.t. candidate's coordinates for C) A1

$\tan C\hat{A}D = \frac{CD}{AC} = 2$ (or ${}^{10}/_{5}$ or equivalent)

(f.t. candidate's derived values for CD and AC) B1

2. (*a*) $$\frac{8-\sqrt{7}}{\sqrt{7}-2}=\frac{(8-\sqrt{7})(\sqrt{7}+2)}{(\sqrt{7}-2)(\sqrt{7}+2)}$$ M1

Numerator: $8\sqrt{7}+16-7-2\sqrt{7}$ A1

Denominator: $7-4$ A1

$$\frac{8-\sqrt{7}}{\sqrt{7}-2}=\frac{6\sqrt{7}+9}{3}=2\sqrt{7}+3$$ (c.a.o.) A1

Special case

If M1 not gained, allow B1 for correctly simplified numerator or denominator following multiplication of top and bottom by $\sqrt{7}-2$

(*b*) $\sqrt{50}=5\sqrt{2}$ B1

$\sqrt{3}\times\sqrt{6}=3\sqrt{2}$ B1

$-\frac{14}{\sqrt{2}}=-7\sqrt{2}$ B1

$\sqrt{50}+(\sqrt{3}\times\sqrt{6})-\frac{14}{\sqrt{2}}=\sqrt{2}$ (c.a.o.) B1

3. $\frac{dy}{dx}=4x+6$ (an attempt to differentiate, at least one non-zero term correct) M1

An attempt to substitute $x=-1$ in candidate's expression for $\frac{dy}{dx}$ m1

Gradient of tangent at $P=2$ (c.a.o.) A1

y-coordinate at $P=3$ B1

Equation of tangent at P: $y-3=2[x-(-1)]$ (or equivalent)

(f.t. one slip provided both M1 and m1 awarded) A1

4. (*a*) (i) $a=-2{\cdot}5$ (or equivalent) B1

$b=1{\cdot}75$ (or equivalent) B1

(ii) Greatest value $=-b$ (or equivalent) B1

(*b*) $x^2-x-7=2x+3$ M1

An attempt to collect terms, form and solve quadratic equation m1

$x^2-3x-10=0\Rightarrow(x-5)(x+2)=0\Rightarrow x=5, x=-2$

(both values, c.a.o.) A1

When $x=5$, $y=13$, when $x=-2$, $y=-1$

(both values f.t. one slip) A1

The line $y=2x+3$ intersects the curve $y=x^2-x-7$ at the points $(-2,-1)$ and $(5, 13)$ (f.t. candidate's points) E1

5.	(*a*)	$y + \delta y = 4(x + \delta x)^2 - 5(x + \delta x) - 3$	B1
		Subtracting y from above to find δy	M1
		$\delta y = 8x\delta x + 4(\delta x)^2 - 5\delta x$	A1
		Dividing by δx and letting $\delta x \to 0$	M1
		$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x} = 8x - 5$ (c.a.o.)	A1
	(*b*)	Required derivative = $7 \times {}^3/_4 \times x^{-1/4} - 2 \times (-4) \times x^{-5}$	B1, B1
6.	(*a*)	An expression for $b^2 - 4ac$, with at least two of a, b, c correct	M1
		$b^2 - 4ac = (2k)^2 - 4(k+1)(k-1)$	A1
		$b^2 - 4ac = 4$ (c.a.o.)	A1
		candidate's value for $b^2 - 4ac > 0$ ($\Rightarrow$ two distinct real roots)	A1
	(*b*)	Finding critical values $x = -2$, $x = {}^3/_5$	B1
		$-2 \le x \le {}^3/_5$ **or** ${}^3/_5 \ge x \ge -2$ **or** $[-2, {}^3/_5]$ **or** $-2 \le x$ and $x \le {}^3/_5$ **or** a correctly worded statement to the effect that x lies between -2 and ${}^3/_5$ (both inclusive) (f.t. critical values ± 2, $\pm {}^3/_5$)	B2
		Note: $-2 < x < {}^3/_5$, $-2 \le x, x \le {}^3/_5$ $-2 \le x \;\; x \le {}^3/_5$ $-2 \le x$ or $x \le {}^3/_5$ all earn B1	
7.	(*a*)	$\left(x + \frac{2}{x}\right)^4 = x^4 + 4x^3\left(\frac{2}{x}\right) + 6x^2\left(\frac{2}{x}\right)^2 + 4x\left(\frac{2}{x}\right)^3 + \left(\frac{2}{x}\right)^4$	
		(three terms correct)	B1
		(all terms correct)	B2
		$\left(x + \frac{2}{x}\right)^4 = x^4 + 8x^2 + 24 + \frac{32}{x^2} + \frac{16}{x^4}$ (three terms correct)	B1
		(all terms correct)	B2
		(– 1 for incorrect further 'simplification')	
	(*b*)	A correct equation in n, including ${}^nC_2 = 55$	M1
		$n = 11, -10$ (c.a.o.)	A1
		$n = 11$ (f.t. $n = 10$ from $n = -11, 10$)	A1

8. (*a*) Use of $f(-1) = -3$ M1

$-a - 1 + 6 + 5 = -3 \Rightarrow a = 2$

A1

(*b*) Attempting to find $f(r) = 0$ for some value of r M1

$f(2) = 0 \Rightarrow x - 2$ is a factor A1

$f(x) = (x - 2)(8x^2 + ax + b)$ with one of a, b correct M1

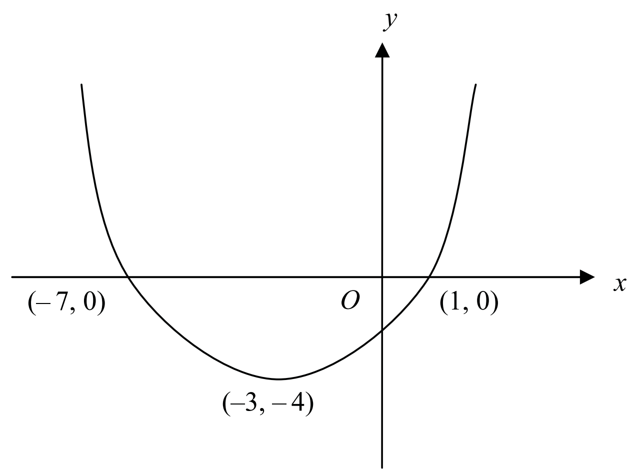
$f(x) = (x - 2)(8x^2 + 2x - 3)$ A1

$f(x) = (x - 2)(4x + 3)(2x - 1)$ (f.t. only $8x^2 - 2x - 3$ in above line) A1

Special case

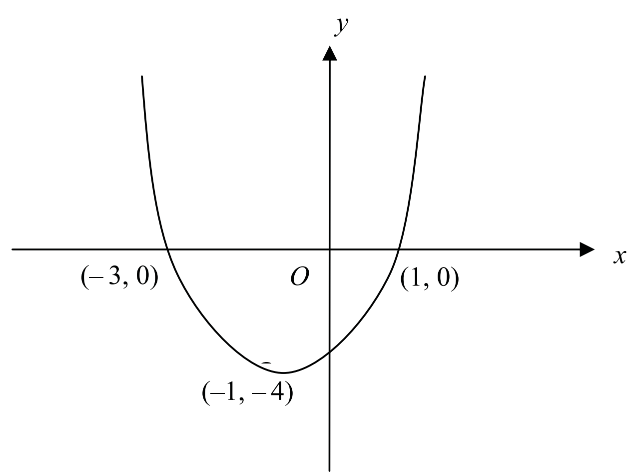
Candidates who, after having found $x - 2$ as one factor, then find one of the remaining factors by using e.g. the factor theorem, are awarded B1 for final 3 marks

9. (*a*)



Concave up curve and y-coordinate of minimum = – 4	B1
x-coordinate of minimum = – 3	B1
Both points of intersection with x-axis	B1

(*b*)



Concave up curve and y-coordinate of minimum = – 4	B1
x-coordinate of minimum = – 1	B1
Both points of intersection with x-axis	B1

10. (*a*) $\frac{dy}{dx} = 3x^2 - 6x + 3$ B1

Putting derived $\frac{dy}{dx} = 0$ M1

$3(x - 1)^2 = 0 \Rightarrow x = 1$ A1

$x = 1 \Rightarrow y = 6 \Rightarrow$ stationary point is at (1, 6) (c.a.o) A1

(*b*) **Either:**

An attempt to consider value of $\frac{dy}{dx}$ at $x = 1^-$ and $x = 1^+$ M1

$\frac{dy}{dx}$ has same sign at $x = 1^-$ and $x = 1^+ \Rightarrow$ (1, 6) is a point of inflection A1

Or:

An attempt to find value of $\frac{d^2y}{dx^2}$ at $x = 1$, $x = 1^-$ and $x = 1^+$ M1

$\frac{d^2y}{dx^2} = 0$ at $x = 1$ and $\frac{d^2y}{dx^2}$ has different signs at $x = 1^-$ and $x = 1^+$

$\Rightarrow$ (1, 6) is a point of inflection A1

Or:

An attempt to find the value of y at $x = 1^-$ and $x = 1^+$ M1

Value of y at $x = 1^- < 6$ and value of y at $x = 1^+ > 6 \Rightarrow$ (1, 6) is a point of inflection A1

Or:

An attempt to find values of $\frac{d^2y}{dx^2}$ and $\frac{d^3y}{dx^3}$ at $x = 1$ M1

$\frac{d^2y}{dx^2} = 0$ and $\frac{d^3y}{dx^3} \neq 0$ at $x = 1 \Rightarrow$ (1, 6) is a point of inflection A1