Mathematics C1 January 2009

Solutions and Mark Scheme

Final Version

1.	(*a*)	Gradient of $BC = \dfrac{\text{increase in } y}{\text{increase in } x}$	M1
		Gradient of $BC = {}^1/_4$ (or equivalent)	A1
		A correct method for finding the equation of $BC(AD)$ using candidate's gradient for BC	M1
		Equation of BC : $y - 4 = {}^1/_4 (x - 5)$ (or equivalent) (f.t. candidate's gradient for BC)	A1
		Equation of BC : $x - 4y + 11 = 0$ (convincing)	A1
		Use of $m_{AB} \times m_{CD} = -1$	M1
		Equation of AD : $y - (-1) = -4(x - 2)$ (or equivalent) (f.t. candidate's gradient of BC)	A1
		Special case:	
		Verification of equation of BC by substituting coordinates of **both** B and C into the given equation	M1
		Making an appropriate statement	A1
	(*b*)	An attempt to solve equations of BC and AD simultaneously	M1
		$x = 1, y = 3$ (convincing) (c.a.o.)	A1
		Special case	
		Substituting (1, 3) in equations of **both** BC and AD	M1
		Convincing argument that coordinates of D are (1, 3)	A1
	(*c*)	A correct method for finding the length of CD	M1
		$CD = \sqrt{17}$	A1
	(*d*)	A correct method for finding E	M1
		$E(0, 7)$	A1

2. (*a*) $\frac{10\sqrt{3} - 1}{4 - \sqrt{3}} = \frac{(10\sqrt{3} - 1)(4 + \sqrt{3})}{(4 - \sqrt{3})(4 + \sqrt{3})}$ M1

Numerator: $40\sqrt{3} + 10 \times 3 - 4 - \sqrt{3}$ A1

Denominator: $16 - 3$ A1

$\frac{10\sqrt{3} - 1}{4 - \sqrt{3}} = \frac{39\sqrt{3} + 26}{13} = 3\sqrt{3} + 2$ (c.a.o.) A1

Special case

If M1 not gained, allow B1 for correctly simplified numerator or denominator following multiplication of top and bottom by $4 - \sqrt{3}$

(*b*) $(2 + \sqrt{5})(5 - \sqrt{20}) = 10 - 2\sqrt{20} + 5\sqrt{5} - \sqrt{5} \times \sqrt{20}$

(4 terms, at least 3 correct) M1

$\sqrt{20} = 2\sqrt{5}$ B1

$\sqrt{5} \times \sqrt{20} = 10$ B1

$(2 + \sqrt{5})(5 - \sqrt{20}) = \sqrt{5}$ (c.a.o.) A1

Alternative Mark Scheme

$(2 + \sqrt{5})(5 - \sqrt{20}) = (2 + \sqrt{5})(5 - 2\sqrt{5})$ B1

$(2 + \sqrt{5})(5 - 2\sqrt{5}) = 10 - 4\sqrt{5} + 5\sqrt{5} - \sqrt{5} \times 2\sqrt{5}$

(4 terms, at least 3 correct) M1

$\sqrt{5} \times 2\sqrt{5} = 10$ B1

$(2 + \sqrt{5})(5 - \sqrt{20}) = \sqrt{5}$ (c.a.o.) A1

3. (*a*) $\frac{dy}{dx} = 2x - 9$ (an attempt to differentiate, at least one non-zero term correct) M1

An attempt to substitute $x = 6$ in candidate's expression for $\frac{dy}{dx}$ m1

Gradient of tangent at $P = 3$ (c.a.o.) A1

Equation of tangent at P: $y - (-5) = 3(x - 6)$ (or equivalent)

(f.t. candidate's value for $\frac{dy}{dx}$ provided both M1 and m1 awarded) A1

(*b*) Use of gradient of tangent at $Q \times {}^{1}/_{7} = -1$ M1

Equating candidate's expression for $\frac{dy}{dx}$ and candidate's value for gradient of tangent at Q m1

$2x - 9 = -7 \Rightarrow x = 1$ (f.t. candidate's expression for $\frac{dy}{dx}$) A1

4. $a = 3$ B1

$b = -2$ B1

$c = 5$ B1

A positive quadratic graph M1

Minimum point $(-b, c)$ A1

5. An expression for $b^2 - 4ac$, with at least two of a, b, c correct — M1
$b^2 - 4ac = 8^2 - 4 \times (3k - 2) \times k$ — A1
Putting $b^2 - 4ac < 0$ — m1
$3k^2 - 2k - 16 > 0$ (convincing) — A1
Finding critical points $k = -2$, $k = {}^8/_3$ — B1
A statement (mathematical or otherwise) to the effect that
$k < -2$ or ${}^8/_3 < k$ (or equivalent) (f.t. candidate's critical points) — B2
Deduct 1 mark for each of the following errors
the use of non-strict inequalities
the use of the word 'and' instead of the word 'or'

6. (*a*) $(a + b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$ (–1 for each error)
(–1 for any subsequent 'simplification') — B2

(*b*) An expression containing $k \times (1/4)^2 \times (2x)^3$, where k is an integer $\neq 1$ and is either the candidate's coefficient for the a^2b^3 term in (*a*) or is derived from first principles — M1
Coefficient of $x^3 = 5$ (c.a.o.) — A1

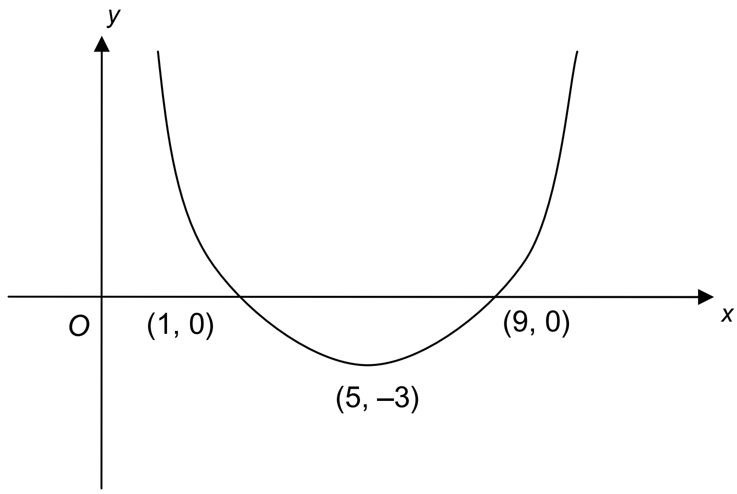
7. (*a*) An attempt to calculate $3^3 - 17$ — M1
Remainder = 10 — A1

(*b*) Attempting to find $f(r) = 0$ for some value of r — M1
$f(2) = 0 \Rightarrow x - 2$ is a factor — A1
$f(x) = (x - 2)(6x^2 + ax + b)$ with one of a, b correct — M1
$f(x) = (x - 2)(6x^2 + 5x - 4)$ — A1
$f(x) = (x - 2)(3x + 4)(2x - 1)$ (f.t. only $6x^2 - 5x - 4$ in above line) — A1
Roots are $x = 2, -{}^4/_3, {}^1/_2$ (f.t. for factors $3x \pm 4$, $2x \pm 1$) — A1
Special case
Candidates who, after having found $x - 2$ as one factor, then find one of the remaining factors by using e.g. the factor theorem, are awarded B1

8. (*a*) $y + \delta y = 7(x + \delta x)^2 + 5(x + \delta x) - 2$ — B1
Subtracting y from above to find δy — M1
$\delta y = 14x\delta x + 7(\delta x)^2 + 5\delta x$ — A1
Dividing by δx and letting $\delta x \to 0$ — M1
$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x} = 14x + 5$ (c.a.o.) — A1

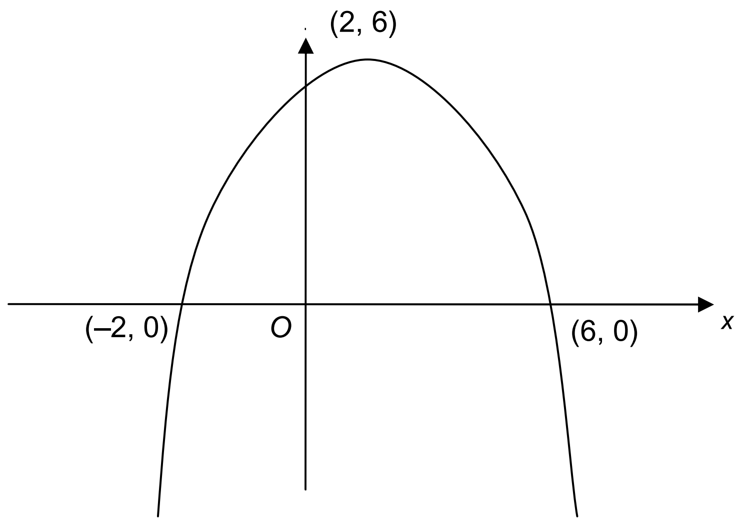
(*b*) Required derivative $= 2 \times (-3) \times x^{-4} + 5 \times ({}^2/_3) \times x^{-1/3}$ — B1, B1

9. (*a*)



Concave up curve and y-coordinate of minimum = –3 B1
x-coordinate of minimum = 5 B1
Both points of intersection with x-axis B1

(*b*)



Concave down curve and x-coordinate of maximum = 2 B1
y-coordinate of maximum = 6 B1
Both points of intersection with x-axis B1

10. (*a*) $\frac{dy}{dx} = 3x^2 + 6x - 9$ B1

Putting derived $\frac{dy}{dx} = 0$ M1

$x = -3, 1$ (both correct) (f.t. candidate's $\frac{dy}{dx}$) A1

Stationary points are (–3, 14) and (1, –18) (both correct) (c.a.o) A1

A correct method for finding nature of stationary points yielding

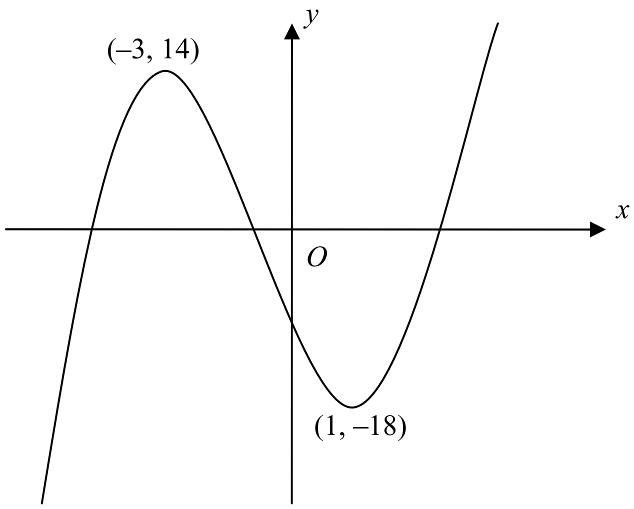
either (–3, 14) is a maximum point

or (1, –18) is a minimum point (f.t. candidate's derived values) M1

Correct conclusion for other point

(f.t. candidate's derived values) A1

(*b*)



Graph in shape of a positive cubic with two turning points M1

Correct marking of both stationary points

(f.t. candidate's derived maximum and minimum points) A1

(*c*) A statement identifying the number of roots as the number of times the curve crosses the x-axis (any curve) M1

Correct interpretation of the number of roots from the candidate's **cubic** graph. A1