Mathematics C1 May 2008

Solutions and Mark Scheme

1. (*a*) Gradient of $AB = \dfrac{\text{increase in } y}{\text{increase in } x}$ M1

Gradient of $AB = -{}^1/_2$ (or equivalent) A1

(*b*) A correct method for finding the equation of AB using the candidate's value for the gradient of AB. M1

Equation of AB : $y - 4 = -{}^1/_2\,[x - (-7)]$ (or equivalent) A1

(f.t. the candidate's value for the gradient of AB)

Equation of AB : $x + 2y - 1 = 0$

(f.t. one error if both M1's are awarded) A1

(*c*) A correct method for finding the length of AB M1

$AB = \sqrt{125}$ A1

(*d*) A correct method for finding E M1

$E(-2,\ 1{\cdot}5)$ A1

(*e*) **Either:**

An attempt to find the gradient of a line perpendicular to AB using the fact that the product of the gradients of perpendicular lines = –1. M1

An attempt to find the gradient of the line passing through C and D M1

$2 = \dfrac{1 - (-15)}{6 - k}$ (Equating expressions for gradient) M1

$k = -2$ (f.t. candidate's gradient of AB) A1

Or:

An attempt to find the gradient of a line perpendicular to AB using the fact that the product of the gradients of perpendicular lines = –1. M1

An attempt to find the equation of line perpendicular to AB passing through C(or D) M1

$-15 - 1 = 2(k - 6)$

(substituting coordinates of unused point in the equation) M1

$k = -2$ (f.t. candidate's gradient of AB) A1

2. (*a*) **Either:**

$\sqrt{75} = 5\sqrt{3}$ B1

$\dfrac{9}{\sqrt{3}} = 3\sqrt{3}$ B1

$\sqrt{6} \times \sqrt{2} = 2\sqrt{3}$ B1

$\sqrt{75} - \dfrac{9}{\sqrt{3}} + (\sqrt{6} \times \sqrt{2}) = 4\sqrt{3}$ (c.a.o.) B1

Or:

$\sqrt{75} = \dfrac{15}{\sqrt{3}}$ B1

$\sqrt{6} \times \sqrt{2} = \dfrac{6}{\sqrt{3}}$ B1

$\sqrt{75} - \dfrac{9}{\sqrt{3}} + (\sqrt{6} \times \sqrt{2}) = \dfrac{12}{\sqrt{3}}$ B1

$\sqrt{75} - \dfrac{9}{\sqrt{3}} + (\sqrt{6} \times \sqrt{2}) = 4\sqrt{3}$ (c.a.o.) B1

(*b*) $\dfrac{5\sqrt{5} - 2}{4 + \sqrt{5}} = \dfrac{(5\sqrt{5} - 2)(4 - \sqrt{5})}{(4 + \sqrt{5})(4 - \sqrt{5})}$ M1

Numerator: $20\sqrt{5} - 25 - 8 + 2\sqrt{5}$ A1

Denominator: $16 - 5$ A1

$\dfrac{5\sqrt{5} - 2}{4 + \sqrt{5}} = 2\sqrt{5} - 3$ (c.a.o.) A1

Special case

If M1 not gained, allow B1 for correctly simplified numerator or denominator following multiplication of top and bottom by $4 + \sqrt{5}$

3. $\dfrac{dy}{dx} = 6x - 8$

(an attempt to differentiate, at least one non-zero term correct) M1

An attempt to substitute $x = 2$ in candidate's expression for $\dfrac{dy}{dx}$ m1

Value of $\dfrac{dy}{dx}$ at $P = 4$ (c.a.o.) A1

Gradient of normal = $\dfrac{-1}{\text{candidate's value for } \frac{dy}{dx}}$ m1

y-coordinate of $P = 3$ B1

Equation of normal to C at P: $y - 3 = -{}^{1}/_{4}(x - 2)$ (or equivalent) A1

(f.t. **one** slip in **either** candidate's value for $\dfrac{dy}{dx}$ **or** candidate's value for the y-coordinate at P provided M1and both m1's awarded)

4. (*a*) $y + \delta y = 5(x + \delta x)^2 + 3(x + \delta x) - 4$ B1

Subtracting y from above to find δy M1

$\delta y = 10x\delta x + 5(\delta x)^2 + 3\delta x$ A1

Dividing by δx and letting $\delta x \to 0$ M1

$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x} = 10x + 3$ (c.a.o.) A1

(*b*) $\frac{dy}{dx} = -8 \times x^{-2} + 3 \times \frac{1}{2} \times x^{-1/2}$ B1, B1

Either $4^{-2} = \frac{1}{16}$ **or** $4^{-1/2} = \frac{1}{2}$ B1

$\frac{dy}{dx} = \frac{1}{4}$ (c.a.o) B1

5. (*a*) $a = 3$ B1

$b = -13$ B1

(*b*) $2b$ on its own or least (minimum) value = $2b$, with correct explanation or no explanation B1

$x = -a$ B1

Note: Candidates who use calculus are awarded B0, B0

6. $(5 + 2x)^3 = 125 + 150x + 60x^2 + 8x^3$

Two terms correct B1

Three terms correct B1

All 4 terms correct B1

7. (*a*) Use of $f(2) = 0$ M1

$32 + 4p - 22 + q = 0$ A1

Use of $f(-1) = 9$ M1

$-4 + p + 11 + q = 9$ A1

Solving simultaneous equations for p and q M1

$p = -4$, $q = 6$ (convincing) A1

Note:

Candidates who assume $p = -4$, $q = 6$ and then verify that $x - 2$ is a factor and that dividing the polynomial by $x + 1$ gives a remainder of 9 may be awarded M1 A1 M1 A1 M0 A0

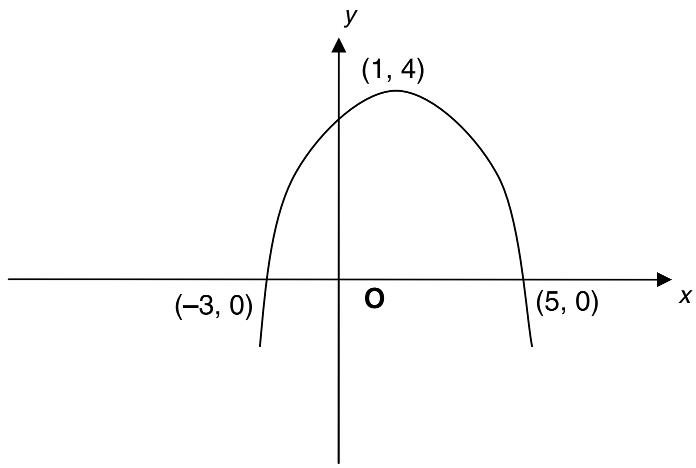
(*b*) $f(x) = (x - 2)(4x^2 + ax + b)$ with one of a, b correct M1

$f(x) = (x - 2)(4x^2 + 4x - 3)$ A1

$f(x) = (x - 2)(2x - 1)(2x + 3)$ A1

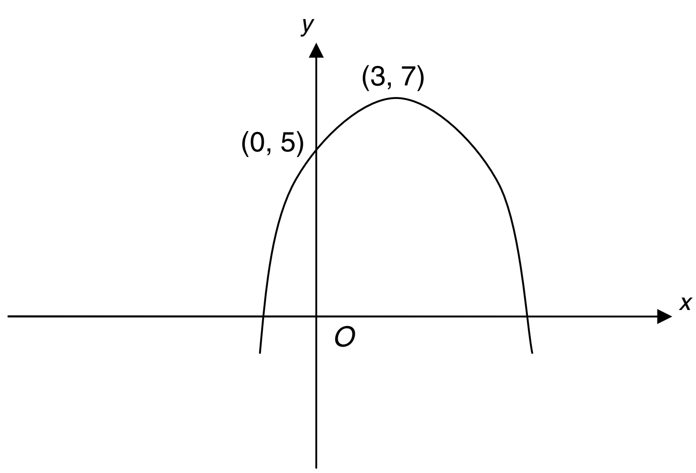
(f.t. only for $f(x) = (x - 2)(2x + 1)(2x - 3)$ from $4x^2 - 4x - 3$)

8. (*a*)



Concave down curve with maximum (1, 4) or (5, 4)	B1
Two of the three points correct	B1
All three points correct	B1

(*b*)



Concave down curve with maximum (3, 7) or (3, 1)	B1
Maximum point (3, 7)	B1
Point of intersection with *y*-axis (0, 5)	B1

9. $\frac{dy}{dx} = -6x^2 + 6x + 12$ B1

Putting derived $\frac{dy}{dx} = 0$ M1

$x = 2, -1$ (both correct) (f.t. candidate's $\frac{dy}{dx}$) A1

Stationary points are (2, 15) and (–1, –12) (both correct) (c.a.o) A1

A correct method for finding nature of stationary points M1

(2, 15) is a maximum point (f.t. candidate's derived values) A1

(–1, –12) is a minimum point (f.t. candidate's derived values) A1

10. (*a*) Finding critical points $x = -1{\cdot}5$, $x = 3$ B1

A statement (mathematical or otherwise) to the effect that

$x \le -1{\cdot}5$ or $3 \le x$ (or equivalent)

(f.t. candidate's critical points) B2

Deduct 1 mark for each of the following errors

the use of strict inequalities

the use of the word 'and' instead of the word 'or'

(*b*) (i) An expression for $b^2 - 4ac$, with $b = (-)6$ and at least one of *a* or *c* correct M1

$b^2 - 4ac = [(-)6]^2 - 4 \times 3 \times m$ A1

$b^2 - 4ac < 0$ m1

$m > 3$ (c.a.o.) A1

(ii) $3x^2 - 4x + 7 = 2x + k$ M1

No points of intersection $\Leftrightarrow 3x^2 - 6x + (7 - k) = 0$ has no real roots (allow one slip in quadratic) m1

$4 > k$ A1

(if candidate uses (*b*)(i), f.t. candidate's inequality for *m*)