M3

Q	Solution	Mark	Notes
1(a)	N2L $\frac{27000}{(t+3)^2} = 600a$	M1	+/-, no additional terms
	$\frac{45}{(t+3)^2} = \frac{dv}{dt}$	m1	use of dv/dt
	$v = -\frac{45}{(t+3)}$ (+ C)	A1	k/(t+3)
		A1	completely correct
	When t = 0, v = 0	m1	use of initial conditions
	C = 15		
	$v = 15 - \frac{45}{(t+3)}$	A1	
	As $t \to \infty$, $v \to 15$	A1	ft similar expression
1(b)	$v = \frac{dx}{dt} = 15 - \frac{45}{(t+3)}$	M1	
	x = 15t – 45 ln(t + 3) (+ C)	A1	ft similar expressions
	t = 0, x = 0 C = 45 ln3	A1	ft
	$x = 15t + 45\ln\left(\frac{3}{t+3}\right)$		
	When t = 6 $x = 90 + 45\ln\left(\frac{3}{9}\right)$	m1	
	x = 90 – 45 ln(3)		
	x = <u>40.56 (m)</u>	A1	cao

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2(a).	Using $v^2 = \omega^2(a^2 - x^2)$	M1	used
	$0.09 \times 3 = \omega^2(a^2 - 0.6^2)$	A1	
	$0.04 \times 5 = \omega^2(a^2 - 0.8^2)$	A1	
	$0.07 = 0.28\omega^2$	m1	
	$\omega = 0.5$		
	$0.2 = 0.25(a^2 - 0.64)$		
	$a = 1.2$	A1	
	Period $= \frac{2\pi}{\omega}$	M1	used
	Period $= \underline{4\pi}$	A1	
2(b)	$\ddot{x} = -\omega^2 x$	M1	used
	$\lvert\ddot{x}\rvert = 0.5^2 \times 0.6$		
	$\lvert\ddot{x}\rvert = \underline{0.15\ (ms^{-2})}$	A1	
2(c)	$x = 1.2\sin(0.5t)$	M1	used, accept cos
	At A, $0.6 = 1.2\sin(0.5t)$		
	$t = 2\sin^{-1}(0.5) = 1.0472$	A1	or 2.0944
	At B, $0.8 = 1.2\sin(0.5t)$		
	$t = 2\sin^{-1}(0.667) = 1.4595$	A1	or 1.6821
	Required $t = 1.4595 - 1.0472$		
	Required $t = \underline{0.412\ (s)}$	A1	cao
2(d)	$x = a\sin(\omega t)$		
	$x = 1.2\sin(0.5t)$	M1	
	$x = 1.2\sin(0.5 \times 2\pi/3)$		
	$x = \underline{1.0392\ (m)}$	A1	
2(e)	$v = a\omega\cos(\omega t)$	M1	oe
	$v = 1.2 \times 0.5\cos(0.5t)$		
	$v = 0.6\cos(0.5t)$	A1	
	When $t = 2\pi/3$, $v = 0.6\cos(0.5\times2\pi/3)$		
	$v = 0.6\cos(\pi/3)$		
	$v = \underline{0.3\ (ms^{-1})}$	A1	cao

Q	Solution	Mark	Notes
3.	Auxiliary equation		
	$2m^2 + 5m + 2 = 0$	B1	cao
	$(2m + 1)(m + 2) = 0$		
	$m = -0.5, -2$	B1	cao
	CF is $x = Ae^{-0.5t} + Be^{-2t}$	B1	ft solutions for m
	For PI, try $x = at + b$	M1	
	$\frac{dx}{dt} = a$		
	$5a + 2(at + b) = 6t + 5$	A1	
	Comparing coefficients	m1	
	$2a = 6$		
	$a = 3$		
	$15 + 2b = 5$		
	$b = -5$	A1	both answers cao
	General solution is		
	$x = Ae^{-0.5t} + Be^{-2t} + 3t - 5$	B1	ft CF and PI
	When $t = 0, x = 3$	M1	use of conditions in GS
	$3 = A + B - 5$		
	$A + B = 8$		
	$\frac{dx}{dt} = -0.5Ae^{-0.5t} - 2Be^{-2t} + 3$	B1	ft similar expressions
	When $t = 0, \frac{dx}{dt} = 2$		
	$2 = -0.5A - 2B + 3$		
	$0.5A + 2B = 1$		
	$A + 4B = 2$		
	$A + B = 8$		
	$3B = -6$		
	$B = \underline{-2}$	A1	cao
	$A = \underline{10}$	A1	cao

Q	Solution	Mark	Notes
4(a)	N2L F = ma	M1	used, no extra term
	$\frac{4}{2x+1} = 0\cdot 5v\frac{dv}{dx}$	m1	use of vdv/dx
	$\int \frac{8}{2x+1}dx = \int v\,dv$	M1	separating variables
	$4\ln\lvert 2x+1\rvert = \frac{1}{2}v^2 + C$	A1	kln(2x+1)
		A1	all correct
	$v^2 = 8\ln\lvert 2x+1\rvert + C$		
	When $x = 3$, $v = 4$	m1	
	$16 = 8\ln 7 + C$		
	$C = 16 - 8\ln 7$	A1	ft kln(2x+1) + C
	$v^2 = 8\ln\left\lvert\frac{2x+1}{7}\right\rvert + 16$		
	When $x = 10$ $v^2 = 8\ln\left\lvert\frac{2\times 10+1}{7}\right\rvert + 16$		
	$v^2 = 8\ln 3 + 16$		
	$v = \underline{4.98\ (\text{ms}^{-1})}$	A1	cao
4(b)	$v = 6,\ 6^2 = 8\ln\left\lvert\frac{2x+1}{7}\right\rvert + 16$	M1	allow similar expressions
	$\ln\left\lvert\frac{2x+1}{7}\right\rvert = \frac{20}{8}$		
	$2x + 1 = 7e^{5/2}$	m1	correct inversion
	$x = 0.5[7e^{5/2} - 1]$		
	$x = \underline{42.1\ (\text{m})}$	A1	cao

Q	Solution	Mark	Notes
5.	Using v = u + at with u=0, a=(±)9.8, t=2.5	M1	
	$v = 9.8 \times 0.5$		
	$v = 4.9\ ms^{-1}$	A1	
	Impulse = Change in momentum	M1	used
	For A $J = 5v$	B1	
	For B $J = 2 \times 4.9 - 2v$	A1	
	Solving $5v = 9.8 - 2v$	m1	
	$7v = 9.8$		
	$v = \underline{1.4\ (ms^{-1})}$	A1	cao
	$J = 5 \times 1.4$		
	$J = \underline{7\ (Ns)}$	A1	cao

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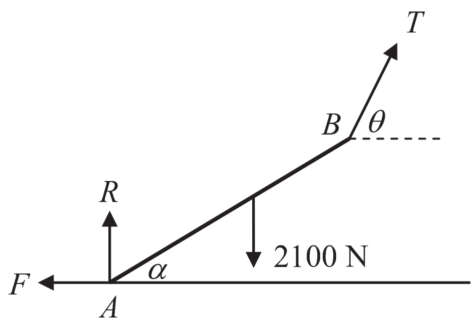
6.(a)			
	$F = \mu R = \frac{3}{4}R$	M1	
	Moments about B	M1	dim correct equation, 3 terms, perp distance
	$R \times 2\cos\alpha + F \times 2\sin\alpha = 2100 \times 1\cos\alpha$	A3	-1 each error
	$R \times 2 \times \frac{12}{13} + \frac{3}{4}R \times 2 \times \frac{5}{13} = 2100 \times \frac{12}{13}$		
	$24R + \frac{15}{2}R = 25200$		
	$R = \underline{800\ (N)}$	A1	cao
6(b)	Resolve vertically	M1	
	$T\sin\theta = 2100 - R$	A1	
	$T\sin\theta = 1300$		
	Resolve horizontally	M1	
	$T\cos\theta = F$	A1	
	$T\cos\theta = \frac{3}{4} \times 800$		
	$T\cos\theta = 600$		
	$T = \sqrt{1300^2 + 600^2}$	m1	oe
	$T = \underline{1432\ (N)}$	A1	cao
	$\theta = \tan^{-1}\left(\frac{1300}{600}\right)$	m1	oe
	$\theta = \underline{65.2°}$	A1	cao