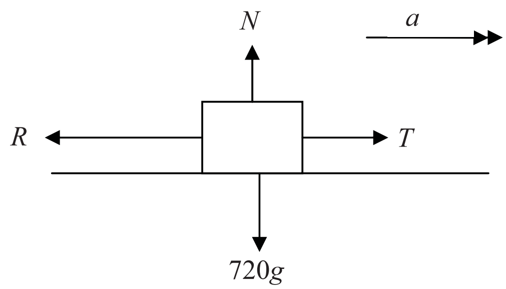
Mathematics M3

1.



(a) Use of $T = \dfrac{P}{v}$ M1

$T = \dfrac{81 \times 1000}{v}$ si A1

Apply N2L to car dim correct equation M1

$T - R = ma$

$\dfrac{81000}{v} - 90v = 720\dfrac{\mathrm{d}v}{\mathrm{d}t}$ A1

Divide by 90 and multiply by v throughout

$900 - v^2 = 8v\dfrac{\mathrm{d}v}{\mathrm{d}t}$ A1

(b) Attempt to separate variables M1

$\displaystyle\int \frac{8v}{900 - v^2}\,\mathrm{d}v = \int \mathrm{d}t$ A1

Integrating

$-4 \ln|900 - v^2| = t + C$ correct ln term A1

all correct A1

$t = -4 \ln|900 - v^2| - C$

Required time $= \left[-4\ln|900 - v^2|\right]_5^{20}$ subtraction of t values M1

correct limits oe A1

$= 4\left[\ln\left(\dfrac{900-25}{900-400}\right)\right]$

$= 4\left[\ln\left(\dfrac{875}{500}\right)\right]$

$= 4\ln(1.75)$

$= \underline{2.24\ (\text{s})}$ cao A1

2. (a) At equilibrium $12g = \dfrac{\lambda \times 0 \cdot 05}{0 \cdot 75}$ use of Hook's Law M1

$\lambda = \underline{1764 \text{ (N)}}$ A1

(b) Consider a displacement x from the equilibrium position.

Apply N2L $12g - T = 12\ddot{x}$ M1

$$12g - \frac{\lambda(0 \cdot 05 + x)}{0 \cdot 75} = 12\ddot{x}$$ ft λ A1

$$\ddot{x} = -(14)^2 x$$

Therefore is SHM (with $\omega = 14$). A1

Amplitude $= \underline{0.05 \text{ (m)}}$ B1

Period $= \dfrac{2\pi}{\omega} = \dfrac{\pi}{7}$ s B1

(c) Maximum speed $= a\omega$ used M1

$= 0.05 \times 14$

$= \underline{0.7 \text{ (ms}^{-1})}$ ft a A1

(d) Use of $v^2 = \omega^2(a^2 - x^2)$ with $\omega = 14$, $a = 0.05$(c), $x = 0.03$ M1

$v^2 = 14^2(0.05^2 - 0.03^2)$ ft a A1

$= 14^2 \times 0.04^2$

$v = \underline{0.56 \text{ (ms}^{-1})}$ cao A1

(e) Displacement from Origin $= x$

$x = (-)0.05\cos(14t)$ M1

When $t = 1.6$

$x = (-)\ 0.05\cos(14 \times 1.6)$ ft a A1

$x = (-)\underline{0.046 \text{ (m)}}$ cao A1

3. Auxiliary equation $4m^2 - 12m + 9 = 0$ B1

$(2m - 3)^2 = 0$

$m = 1.5$ (twice) B1

Complementary function $x = (A + Bt)e^{1.5t}$ ft B1

For PI, try $x = at + b$, $\frac{dx}{dt} = a$ M1

$-12a + 9(at + b) = 18t - 87$ A1

$9a = 18$ comparing coefficients m1

$a = 2$

$-24 + 9b = -87$

$b = -7$ both A1

General solution $x = (A + Bt)e^{1.5t} + 2t - 7$ ft B1

Use of initial conditions $t = 0$, $x = 5$, $\frac{dx}{dt} = 10$ in general solution M1

$A - 7 = 5$

$A = 12$ cao A1

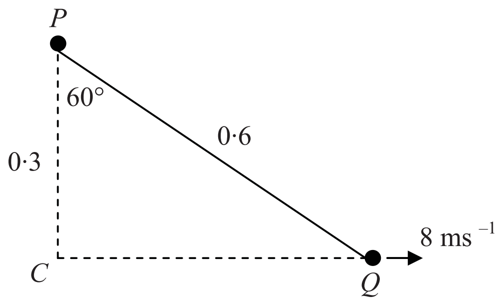
$\frac{dx}{dt} = (A + Bt)(1.5)e^{1.5t} + Be^{1.5t} + 2$ correct diff. ft B1

$1.5A + B + 2 = 10$

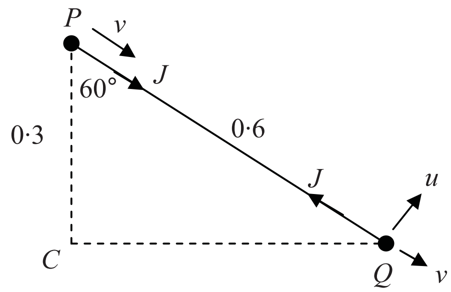
$B = -10$ cao A1

$\underline{x = (12 - 10t)e^{1.5t} + 2t - 7}$

4. When the string jerks tight, each particle begins to move in direction PQ with equal speeds v.



Just before jerk



Just after jerk

$$\cos \angle CPQ = \frac{1}{2}$$

$$\sin \angle CPQ = \frac{\sqrt{3}}{2}$$ si B1

Use of impulse = change in momentum M1

Applied to P $J = 3v$ B1

Applied to Q $J = 5 \times 8\sin 60° - 5v$ A1

Attempt to solve simultaneously m1

$$3v = 40 \times \frac{\sqrt{3}}{2} - 5v$$

$$v = \frac{5\sqrt{3}}{2} = \underline{4.33\ (\text{ms}^{-1})}$$ cao A1

Speed of particle P is 4.33 ms^{-1}.

Magnitude of impulsive tension $= J = 3v$

$$= \frac{15\sqrt{3}}{2} = \underline{12.99\ (\text{Ns})}$$ cao A1

units B1

Perpendicular to PQ, there is no impulse

Speed of particle Q perpendicular to PQ = $8\cos 60° = 4$ ms^{-1} B1

Speed of particle Q = $$\sqrt{4^2 + \left(\frac{5\sqrt{3}}{2}\right)^2}$$ M1

$$= \underline{5.89\ \text{ms}^{-1}}$$ cao A1

5. (a) Use of N2L M1

$$150g - 10v^2 = 150a$$ A1

$$15g - v^2 = 15v\frac{\mathrm{d}v}{\mathrm{d}s}$$ A1

(b) Attempt to separate variables M1

$$\int \frac{15v\ \mathrm{d}v}{v^2 - 15g} = -\int \mathrm{d}s$$ A1

$$\frac{15}{2}\ln\left|v^2 - 15g\right| = -s\,(+C)$$ correct ln A1

all correct A1

Use of boundary conditions $s = 0,\ v = 30$ m1

$$\frac{15}{2}\ln\left|900 - 15g\right| = C$$

$$s = \frac{15}{2}\ln\left|\frac{753}{v^2 - 15g}\right|$$ cao A1

(c) $v = 14$ used M1

$$s = \frac{15}{2}\ln\left(\frac{753}{14^2 - 15\times 9\cdot 8}\right)$$

$s = \underline{20.49}$ cao A1

(d) Removing ln M1

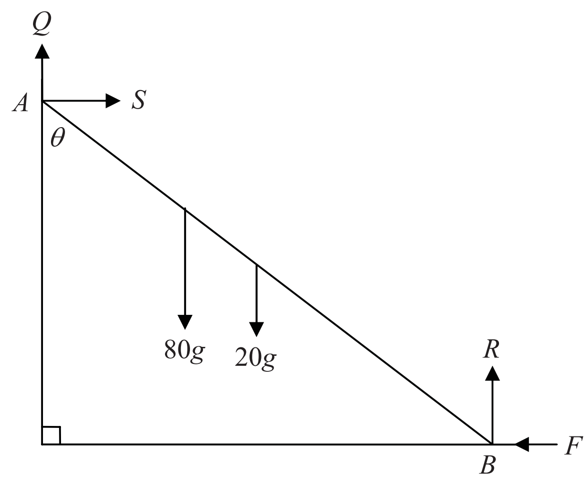
$$\exp\left(\frac{2}{15}s\right) = \frac{753}{v^2 - 15g}$$ ft A1

$$v^2 - 15g = 753\exp\left(-\frac{2}{15}s\right)$$

$$v^2 = 15g + 753\exp\left(-\frac{2}{15}s\right)$$ cao A1

$$v^2 = 147 + 753\exp\left(-\frac{2}{15}s\right)$$

6.



(a) Use of Friction $=$ $\mu \times$ Normal reaction — si M1

$Q = 0.3\,S$ — A1

Attempt at taking mom. about B 4 terms, — dim correct equation M1

$20g \times 2.5 \sin\theta + 80g \times 3\sin\theta = 4S + 3Q$ — -1 each error A2

$294 + 1411.2 = 4S + 0.9S$

$4.9S = 1705.2$

$S = \underline{348\ (\text{N})}$ — cao A1

(b) Resolve vertically — 4 terms, dim correct M1

$Q + R = 80g + 20g$ — A1

$R = 100g - 0.3 \times 348$

$R = 875.6$ N

Resolve horizontally

$F = S\ (= 348)$ — B1

Use of $F \leq \mu R$ — M1

$$\mu \geq \frac{348}{875 \cdot 6} = 0 \cdot 39744$$

$\mu \geq \underline{0.397}$ — cao A1