Mathematics M3 (June 2009)
Final Markscheme

1.(a) Using N2L M1

$$-0.2 - 0.03v = 9\frac{dv}{dt}$$ A1

$$900\frac{dv}{dt} = -(20 + 3v)$$ A1

1.(b) $$900\int\frac{dv}{20+3v} = -\int dt$$ sep. var. M1

$$900.\ \frac{1}{3}\ln(20 + 3v) = -t\ (+ C)$$ A1 A1

When $t = 0$, $v = 20$ used m1

$$C = 300 \ln 80$$

Therefore $t = 300\ln(80) - 300\ln(20 + 3v)$ A1

$$t = 300\ln\left(\frac{80}{20 + 3v}\right)$$

1.(c) When body is at rest, $v=0$ used m1

$t = 300 \ln(80) - 300 \ln(20)$

$= 300\ln(4)$

$= \underline{416 \text{ s}}$ cao A1

2.(a) Amplitude = 24 cm =0.24 m B1

Period = $2\times 4 = 8$ s B1

Therefore $\frac{2\pi}{\omega} = 8$ M1

$$\omega = \frac{\pi}{4}$$ A1

Speed of projection $= a\omega$ used o.e. M1

$$= 0.24 \times \frac{\pi}{4}$$

$= 0.06\pi = \underline{0.188 \text{ ms}^{-1}}$ (= 18.8 cms^{-1}) cao A1

2.(b) $x = 0.24\sin(\frac{\pi}{4}t)$ M1

$0.15 = 0.24 \sin(\frac{\pi}{4}t)$ m1

$t = 0.86$ cao A1

Required time = 8 + 0.86

$= \underline{8.86 \text{ s}}$ ft t and period A1

2.(c) $v = \dfrac{dx}{dt}$ used M1

$v = 0.06\pi\cos(\dfrac{\pi}{4}t)$ ft ω A1

When $t = 1.5$ $v = 0.06\pi\cos(\dfrac{\pi}{4} \times 1.5)$ m1

$v = \underline{0.072\ ms^{-1}}$ (= 7.2 cms^{-1}) cao A1

2.(d) $v^2 = \omega^2 (a^2 - x^2)$ M1

$v^2 = \dfrac{\pi^2}{4^2}\left(0 \cdot 24^2 - 0 \cdot 2^2\right)$ A1

$v = \underline{0.104\ ms^{-1}}$ (=10.4 cms^{-1}) cao A1

3. Apply N2L M1

$180 - 3v^2 = 75a$ A1

$60 - v^2 = 25v\dfrac{dv}{dx}$

$25v\dfrac{dv}{dx} = 60 - v^2$ A1

$25\int\dfrac{v\,dv}{dx} = \int dx$ sep. var. M1

$-\dfrac{25}{2}\ln\left(60 - v^2\right) = x \quad (+C)$ A1 A1

When $x = 0$, $v = 0$ (accept limits) used m1

$-\dfrac{25}{2}\ln(60) = C$ cao A1

$x = \dfrac{25}{2}\ln\left(\dfrac{60}{60 - v^2}\right)$

When $x = 20$

$\ln\left(\dfrac{60}{60 - v^2}\right) = 20 \times \dfrac{2}{25} = 1 \cdot 6$

$\dfrac{60}{60 - v^2} = e^{1.6}$ $x = 20$ and inversion m1

$60 = 60e^{1.6} - e^{1.6}v^2$

$v^2 = \dfrac{60\left(e^{1.6} - 1\right)}{e^{1.6}}$

$v = \underline{6.92\ ms^{-1}}$ cao A1

4.(a) Impulse = change in momentum — used — M1

$1.2 = 3v$

$v = \underline{0.4\ \text{ms}^{-1}}$ — cao — A1

4.(b) For Q $-I = 3v - 3 \times 0.4$ — attempt P or Q — M1

$I = 3v - 1.2$

For P $I = 5v$ — attempt — m1

Both equations correct — A1

Solving simultaneously — m1

$5v = 1.2 - 3v$

$8v = 1.2$

$v = \underline{0.15\ \text{ms}^{-1}}$ — cao — A1

$I = \underline{0.75\ \text{Ns}}$ — cao — A1

4.(c) Loss in energy $= 0.5 \times 3 \times 0.4^2 - 0.5 \times 8 \times 0.15^2$ — ft v's — M1 A1 A1

$= \underline{0.15\ \text{J}}$ — cao — A1

5.(a) N2L $(156 - 52x) - 4v = 2a$ — M1

$2a + 4v + 52x = 156$

$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 26x = 78$ — A1

5.(b) Auxiliary Equation $m^2 + 2m + 26 = 0$ — M1

$m = -1 \pm 5i$ — cao — A1

Complementary function is $x = e^{-t}(A\sin 5t + B\cos 5t)$ — ft m if complex — B1

P. I. try $x = a$

$26a = 78$

$a = 3$ — B1

General solution is $x = e^{-t}(A\sin 5t + B\cos 5t) + 3$ — ft CF + PI — B1

When $t = 0$, $x = 0$ — subst. into GS — m1

$0 = B + 3$

$B = -3$ — ft similar exp. — A1

$\frac{dx}{dt} = -e^{-t}(A\sin 5t + B\cos 5t) + e^{-t}(5A\cos 5t + 5B\sin 5t)$ — ft — B1

When $t = 0$, $\frac{dx}{dt} = 3$ — subst. into "GS" — m1

$3 = 3 + 5A$

$A = 0$ — cao — A1

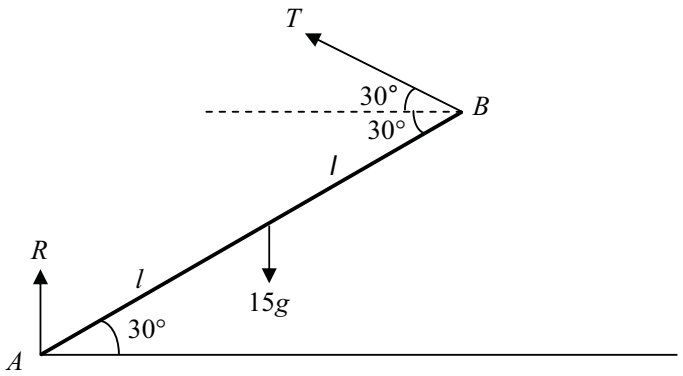
$x = 3 - 3e^{-0.5}\cos 5t$

When $t = 0.5$ — m1

$x = 3 - 3e^{-0.5}\cos(5 \times 0.5)$

$x = \underline{4.46\ \text{m}}$ — cao — A1

6.



Moments about A		dim correct	M1
	$15g \times l \cos 30° = T \times 2l \cos 30°$		A1
	$T = 75g$		
	$T = \underline{75.5 \text{ N}}$		A1
Resolve horizontally			M1
	$T \cos 30° = F$		A1
	$F = 73.5 \cos°30$	subst. for T	m1
	$F = 36.75\sqrt{3}$ N		
Resolve vertically			M1
	$R + T \sin 30° = 15g$		A1
	$R = 15g - 73.5 \times 0.5$	subt. for T	m1
	$R = 110.25$ N		
	$F \leq \mu R$		M1
	$\mu \geq \frac{36 \cdot 75\sqrt{3}}{110 \cdot 25}$	any correct expression	A1
Therefore least value of μ is 0.577 ($\frac{1}{\sqrt{3}}$)		cao	A1