MATHEMATICS M3

1. *(a)* (i) N2L $F = ma$ — used — M1

$400 - 16v^2 = 800a$ — A1

Divide by 800 and using $a = \frac{dv}{dt}$

$\frac{dv}{dt} = \frac{400-16v^2}{800}$

$\frac{dv}{dt} = \frac{25-v^2}{50}$ — convincing — A1

(ii) $50\int\frac{dv}{25-v^2} = \int dt$ — sp.var. — M1

$\frac{50}{2\times 5}\ln\left|\frac{5+v}{5-v}\right| = t + C$ — A2

When $t = 0$, $x = 0$. — m1

$C = 0$ — ft — A1

$t = 5\ln\left|\frac{5+v}{5-v}\right|$

When $v = 2$,

$t = 5\ln\left|\frac{7}{3}\right| = \underline{4.24\text{ s}}$ — cao — A1

(b) $v\frac{dv}{dx} = \frac{25-v^2}{50}$ — M1

$\frac{50}{2}\int\frac{2v}{25-v^2}dv = \int dx$ — m1

$-25\ln\left|25-v^2\right| = x + C$ — A2

When $x = 0$, $v = 0$. — m1

$C = -25\ln|25|$ — ft +/- — A1

When $v = 2$,

$x = 25\ln 25 - 25\ln 21$

$x = \underline{4.36\text{ m}}$ — cao — A1

2. *(a)* Period $= \frac{2\pi}{\omega} = 2\times 4$ M1

$\omega = 0.25\pi$ A1

Max speed $= a\omega = 3\pi$ M1

$a = 3\pi / 0.25\pi$

$a = \underline{12\text{ m}}$ convincing A1

(b)

$x = -12\cos(0.25t)$ +/- M1

When $t = \frac{2}{3}$ $x = -12\cos(0.25\pi \times \frac{2}{3})$ m1

$x = -12 \times \frac{\sqrt{3}}{2} = -6\sqrt{3}$ +/- A1

Distance of P from A when $t = \frac{2}{3} = 12 - 6\sqrt{3} = \underline{1.61\text{ m}}$ cao A1

(c) $v = \frac{dx}{dt}$ used M1

$v = -12 \,.\, -\sin(0.25t) \,.\, (0.25\pi)$

$v = 3\pi\sin(0.25t)$ ft A1

When $t = \frac{2}{3}$, $v = 3\pi\sin(0.25\pi \times \frac{2}{3}) = \underline{1.5\pi\text{ ms}^{-1}}$ cao A1

(d) At X, $x = -5$ M1

$-5 = -12\cos(0.25\pi)$

$t_X = \underline{1.4528\text{ s}}$ ft ω A1

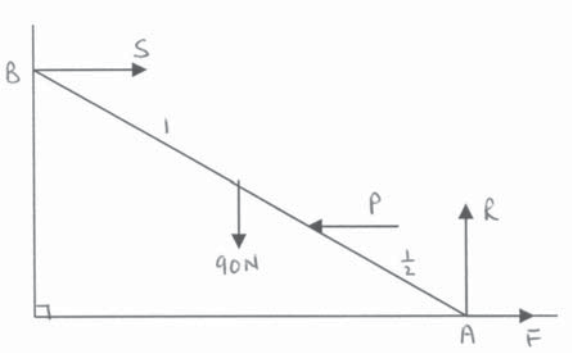
At Y, $x = 5$

$5 = -12\cos(0.25\pi)$

$t_Y = \underline{5.5472\text{ s}}$ ft ω A1

Therefore required time $= t_Y - t_X = 1.0944 = \underline{1.09\text{ s}}$ cao A1

3. *(a)*



B1 B1

(b) Resolve vertically upwards M1

$R = 90$ A1

Resolve horizontally to the right M1

$S + F = P$ ft diagram A1

Moments about A M1

$P \times 0.5 \cos\theta + 90 \sin\theta = S \times 2\cos\theta$ A1 A1

$P + 180 \tan\theta = 4S$

$P - 0.6 \times 90 = S$ $F = \mu R$ B1

Therefore $P + 180 \times 0.8 = 4(P - 54)$ m1

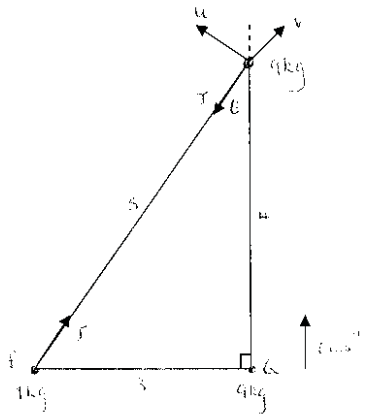
$P + 144 = 4P - 216$

$3P = 360$

$P = \underline{120\text{ N}}$ A1

(c) I have assumed that the ladder is a rigid rod. B1

4.



$\cos\theta = 0.8$,	$\sin\theta = 0.6$	si	B1
$u = 6\sin\theta$			M1
$u = 6 \times 0.6 = 3.6\ \text{ms}^{-1}$		si	A1

Impulse = change in momentum

For P			M1
$J = 7v$			A1
For Q			M1
$9 \times 6\cos\theta - J = 9v$			A1
Adding $54 \times 0.8 = 16v$			m1
$v = 2.7\ \text{ms}^{-1}$			A1
Speed of P = $\underline{2.7\ \text{ms}^{-1}}$			
Speed of Q = $\sqrt{2.7^2 + 3.6^2}$			M1
= $\underline{4.5\ \text{ms}^{-1}}$		ft 2.7(c)	A1
$J = 7 \times 2.7$			
= $\underline{18.9\ \text{Ns}}$		ft 2.7(c)	A1
Required angle α = $\tan^{-1}\left(\frac{u}{v}\right)$			M1
= $\tan^{-1}\left(\frac{3.6}{2.7}\right)$			
= $\underline{53.13^{\circ}}$		ft	A1

5. *(a)* N2L M1

$(6120 - 80t) - (120 + 40v) = 800a$ A1

$6000 - 40v - 80t = 800a$ A1

Divide by 40

$$150 - \frac{dx}{dt} - 2t = 20\frac{d^2x}{dt^2}$$ B1

$$20\frac{d^2x}{dt^2} + \frac{dx}{dt} = 150 - 2t$$ a1

(b) Auxiliary equation $20m^2 + m = 0$ M1

$m = 0, -0.05$ both A1

Complementary Function is $x = A + Be^{-0.05t}$ ft m B1

Particular integral, try $x = at^2 + bt$ M1

$$\frac{dx}{dt} = 2at + b, \quad \frac{d^2x}{dt^2} = 2a$$

$20(2a) + (2at + b) = 150 - 2t$ A1

$2a = -2$ comp.coef. m1

$a = -1$

$-40 + b = 150$

$b = 190$ both cao A1

General solution is $x = A + B^{-0.05t} - t^2 + 190t$ B1

When $t = 0, x = 0 \Rightarrow$ $A + B = 0$ m1

$$\frac{dx}{dt} = -0.05Be^{-0.05t} - 2t + 190$$ ft B1

When $t = 0, \frac{dx}{dt} = 0 \Rightarrow$ $0 = -0.05B + 190$ m1

$B = 3800$

$A = -3800$ both cao A1

Therefore $\underline{x = 3800(e^{-0.05t} - 1) - t^2 + 190t}$