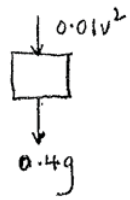
MATHEMATICS M3

1.	*(a)*	$a = v\dfrac{dv}{dx}$	used	M1
		$= \left(\dfrac{B}{x+A}\right)\left[-B(x+A)^{-2}\right]$		A1
		$= \dfrac{-B^2}{(x+A)^3}$		A1
	(b)	$t = 0,\ v = 12,\ x = 0$ $\therefore B = 12A$		M1
		$t = 0,\ a = -16,\ x = 0$ $-B^2 = -16A^3$		A1
		$144A^2 = 16A^3$		
		$A = 9$ $B = 108$ as required	convincing	A1
	(c)	$v = \dfrac{dx}{dt} = \dfrac{108}{x+9}$		M1
		$\int (x+9)dx = 108\int dt$		
		$\dfrac{x^2}{2} + 9x = 108t + C$		A1
		$t = 0,\ x = 0 \Rightarrow C = 0$	f.t. minor error	A1
		$\therefore 216t = x^2 + 18x$ $t = \dfrac{1}{216}x(x+18)$		A1
2.	Auxiliary equation $m^2 + 2m + 10 = 0$			B1
		$m = \dfrac{-2 \pm \sqrt{4-40}}{2}$ $= -1 \pm 3i$		B1
		$\therefore$ C.F. - is $x = e^{-t}(A\sin 3t + B\cos 3t)$		B1
	For P.I. try $x = at + b$			M1
		$\dfrac{dx}{dt} = a$		
	$\therefore\ 2a + 10(at + b) = 5t - 14$			A1
		$10a = 5$	comp. coeff.	M1
		$a = \dfrac{1}{2}$		

$2a + 10b = -14$		
$b = -\frac{3}{2}$	both c.a.o.	A1
$\therefore$ General solution is $x = e^{-t}(A \sin 3t + B \cos 3t) + \frac{1}{2}t - \frac{3}{2}$		B1
When $t = 0$, $x = 4\frac{1}{2}$, $\frac{dx}{dt} = 3\frac{1}{2}$	used	M1
$4\frac{1}{2} = B - \frac{3}{2}$		
$B = 6$	f.t. a.b.	A1
$\frac{dx}{dt} = -e^{-t}(A \sin 3t + B \cos 3t) + e^{-t}(3A \cos 3t - 3B \sin 3t) + \frac{1}{2}$		B1
$3\frac{1}{2} = -B + 3A + \frac{1}{2}$		
$A = 3$	c.a.o.	A1
$\therefore x = 3e^{-t}(\sin 3t + 2\cos 3t) + \frac{1}{2}t - \frac{3}{2}$		

3.



(a) N2L $-0{\cdot}01v^2 - 0{\cdot}4g = 0{\cdot}4a$		M1
$0{\cdot}4\, v\frac{dv}{dx} = -3{\cdot}92 - 0{\cdot}01v^2$	$a = v\frac{dv}{dx}$	A1
$\times 100$ $\quad 40\, v\frac{dv}{dx} = -(392 + v^2)$	convincing	A1
(b) $40\int \frac{v\,dv}{(392 + v^2)} = -\int dx$	sep. var.	M1
$20\ ln\,(392 + v^2) = -x + C$		A1 A1
$t = 0$, $v = 17$, $x = 0$		m1
$\therefore 20\ ln\,(392 + 17^2) = C$		
$C = 20\ ln\,(681)$	f.t. minor error	A1
$x = 20 ln\,(681) - 20\ ln\,(392 + v^2)$		
$= 20\ ln\left(\frac{681}{392 + v^2}\right)$		
At greatest height, $v = 0$		m1
$\therefore x = 20\ ln\left(\frac{681}{392}\right)$		
$= \underline{11.05 \text{ m}}$	c.a.o.	A1
(c) Speed of ball when it returns to O is less than 17 ms^{-1}		B1
because energy used (lost) in overcoming air resistance.		B1

4. *(a)* Period $= \frac{2\pi}{\omega} = 4$ M1

$\omega = \frac{\pi}{2}$ A1

Using $v_{MAX} = a\omega$ with $v_{MAX} = 3\pi$, $\omega = \frac{\pi}{2}$ M1

$3\pi = a \times \frac{\pi}{2}$

$a = \underline{6 \text{ m}}$ c.a.o. A1

(b) Using $v^2 = \omega^2(a^2 - x^2)$ with $\omega = \frac{\pi}{2}$ (c), $a = 6$ (c), $x = 4{\cdot}8$ M1

$v^2 = \frac{\pi^2}{4}(36 - 4{\cdot}8^2)$ f.t. *a, ω* A1

$v = \underline{1{\cdot}8\pi}$

$= \underline{5{\cdot}65 \text{ ms}^{-1}}$ f.t. *a, ω* A1

(c) Let x = distance from O, $y = 0$, p is at O

$x = 6 \sin\left(\frac{\pi}{2}t\right)$ f.t. *a, ω* B1

$4{\cdot}8 = 6 \sin\left(\frac{\pi}{2}t\right)$ M1

$\sin\left(\frac{\pi}{2}t\right) = \frac{4\cdot 8}{6} = 0\cdot 8$

$t = \frac{2}{\pi}\sin^{-1}(0{\cdot}8)$

$= \underline{0{\cdot}59s}$ f.t. *a, ω* A1

(d) Max acceleration when $x = a$ M1

| Max acceleration | $= \omega^2 a$ M1

$= \frac{\pi^2}{4} \times 6$

$= \frac{3\pi^2}{2}$

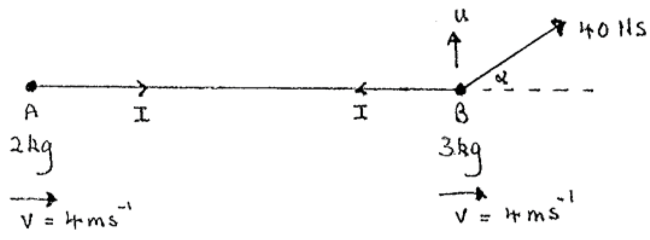
$= \underline{14{\cdot}8 \text{ ms}^{-1}}$ f.t. *a, ω* A1

(e) Distance travelled $= \frac{12}{4}$ oscillations

$= 3 \times (4a)$ M1

$= \underline{72\text{m}}$ f.t. *a* A1

5. *(a)*



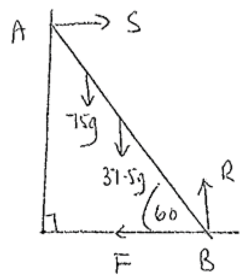
Impulse = change in momentum

Apply to A	I	=	$2v$	M1
		=	2×4	A1
Apply to B	$-\,\text{I}$	=	$-\,40 \cos \alpha + 3\,v$	M1
		=	$-\,40 \cos \alpha + 3 \times 4$	A1
$\therefore$	-8	=	$-\,40 \cos \alpha + 12$	M1
	$40 \cos \alpha$	=	20	
	$\cos \alpha$	=	$\frac{1}{2}$	
	α	=	$\underline{60^{\circ}}$	A1

(b)

	$40 \sin \alpha$	=	$3u$	
	u	=	$40 \times \frac{\sqrt{3}}{2} \times \frac{1}{3}$	
		=	$\frac{20\sqrt{3}}{3}$	A1
Speed of b		=	$\sqrt{\left(\frac{20\sqrt{3}}{3}\right)^2 + 4^2}$	M1
		=	$\underline{12{\cdot}22 \text{ ms}^{-1}}$	A1
	θ	=	$\tan^{-1}\left(\frac{20\sqrt{3}}{3 \times 4}\right)$	M1
		=	$\underline{70{\cdot}89^{\circ}}$	A1

6. *(a)*



Moments about B — dim correct, attempted equation — M1

$37{\cdot}5g \times 4 \cos 60^o + 75g \times x \cos 60^o = S \times 8 \sin 60^o$ — B1 A2

Resolve ↑ $R = 37{\cdot}5\,g + 75\,g$ — M1

$= 112{\cdot}5\,g$

Resolved → $S = F$ — M1

$F = \mu R$ — M1

$\therefore S = \mu.\ 112{\cdot}5g$

Substitute S into moment equation and $\mu = 0{\cdot}25$ — m1

$x\,(75\,g \cos 60^o) = 0{\cdot}25 \times 112{\cdot}5\,g \times 8 \sin 60^o - 37{\cdot}5\,g \times 4 \cos 60^o$ — A1

$$x = \frac{112\cdot 5\sqrt{3} - 75}{37\cdot 5}$$

$= \underline{3{\cdot}196}$ — c.a.o. — A1

(b) Substitute $x = 8$ and $s = \mu.112{\cdot}5g$ (c) into moment equation — M2

$\mu.112{\cdot}5g \times 8 \sin 60^o = 37{\cdot}5g \times 4 \cos 60^o + 75g \times 8 \cos 60^o$ — A1

$$\mu. = \frac{300 + 75}{450\sqrt{3}}$$

$= \underline{0{\cdot}481}$ — c.a.o. — A1

(c) Person modelled as particle/
Ladder modelled as rod — B1